DEVELOPMENT OF FACTUAL WARRANTS
FOR LEFT-TURN CHANNELIZATION THROUGH
DIGITAL COMPUTER SIMULATION

A Summary Report for
the Practicing Traffic Engineer

by

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This report presents a summary of the author's doctoral dissertation by the same title and includes a set of design charts suitable for use by the practicing traffic engineer. The author wishes to express his sincere appreciation to the Institute of Traffic Engineers for their sponsorship of field studies and data analyses conducted in this research.

Although this document constitutes the final report to the Institute of Traffic Engineers, it does not mean the end of the author's efforts to refine his simulation model and expand the scope of his results. Recent evaluations of the simulation model described in this report have been performed by the Traffic Research Corporation, and these revealed a few minor flaws in the model programming. Although these are in the process of being corrected at the time of this writing, it was deemed necessary that the subject project be terminated before "total debugging" could be completed. Should additional useful results be determined they will be made available to the traffic engineering profession through the Institute of Traffic Engineers. (The simulation results reported herein should not be affected greatly by any of the modifications being made.)

The author further wishes to acknowledge Prof. Charles J. Keese and other staff members of the Texas Transportation Institute for their helpful counsel during this research. He also wishes to thank TTI as well as several Texas state and city traffic engineers for their assistance in finding suitable study sites.
The author is presently an associate professor in the Department of Civil Engineering at Louisiana State University, Baton Rouge, Louisiana.
ABSTRACT


The presence of left-turning vehicles is one of the principal factors tending to reduce the capacity of an approach to a signalized intersection. Furthermore, as drivers suffer longer delays at an intersection, they may become impatient and make maneuvers that are hazardous.

In order to increase the capacity and to diminish the hazard at a given location, a decision must be made to, a) either prohibit left turns or, b) to redesign the intersection providing separate lanes for left turning vehicles. Prohibiting turns is not always the best solution as it may only transfer the problem elsewhere in the street system. In redesigning the intersection, the traffic engineer has previously had to rely on qualitative warrants or rules-of-thumb to decide on left-turn channelization. The need for quantitative or factual warrants to replace rules-of-thumb stimulated the research described in this report.

The operation of traffic through a signalized intersection has been shown to consist of several complex phenomena. The principal characteristics required to describe this operation include vehicle arrival distributions, vehicle approach speeds, lane change desires and gap requirements for this maneuver, acceleration and deceleration characteristics near the intersection, car-following considerations, driver response to the yellow phase of a traffic signal, starting headways for vehicles
in a stopped queue, and the left-turn driver's required gap in the opposing stream(s) of traffic. A review of the literature showed that, with the exception of the last characteristic, most of these characteristics could be adequately described for digital computer simulation. Simulation was considered the only practical way to study the effect of left turns on traffic operation through such an intersection.

Field studies utilizing time-lapse photography provided a more comprehensive description and quantification of the nature of left turn movements at a multilane signalized intersection than has heretofore been published. The characteristics of left-turn movements so elucidated include the percentage of vehicles that "jump-the-gun" at the beginning of the green phase, the distributions of acceptable gaps for left turns initiated without stopping and from stopped positions in the intersection, and the variation of gap acceptance with the type of gaps formed in the opposing traffic streams.

A simulation model representing the operation of traffic on a four-lane, two-way street at a fixed-time signalized intersection was successfully formulated and programmed for operation on the IBM 7040 digital computer. Although some features of this model are similar to previously reported models, the distinguishing feature of this model is the detailed left-turn sub-routine which incorporates all of the characteristics defined by this research.

Through replication of simulation procedures for several combinations of traffic parameters, it was possible to develop probability curves for determining the level of delay likely to occur for a given set of conditions (approach and opposing traffic volumes, percent left
turns and traffic signal timing). In this research these variables had the following range of values:

1. Traffic Volumes - 300 to 800 vehicles per hour
2. Percent Left Turns - 10 to 30% of inside lane volume
3. Signal Cycle Length - 50 to 90 seconds
4. Green time to Cycle Length Ratio - 0.4 to 0.6

Delay in terms of the percent of vehicles in the inside approach lane delayed more than one signal cycle length was used. By selecting the desired "level-of-service," or level of delay to be permitted, a highway engineer can utilize the probability charts presented in this report to aid in answering the following questions:

1. When should a separate left-turn lane be provided?
2. When should a separate phase be provided for a channelized left-turn lane?
3. How long should a separate left-turn lane be made for storage purposes?

It is recommended in this report that the simulation work be extended to involve a greater range of the variables studied. The method should also be applied to two-lane streets and tee-type intersections.
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INTRODUCTION

The Left-Turn Problem

A signalized intersection is one of the most critical elements of an urban street system as the traffic capacity of any street is limited by the signalized intersection with the lowest capacity. One of the principal factors tending to lower the capacity of such an intersection is the presence of left-turning vehicles.

When traffic is operating through a signalized urban intersection under low volume conditions, the presence of a left-turn vehicle is of little consequence. The driver of such a vehicle on any given intersection approach may be momentarily delayed by a few opposing vehicles, but in a relatively few seconds he and all other drivers desiring to pass through the intersection will be accommodated during the green time available to that approach. However, as the traffic volume on an intersection approach nears capacity, there are fewer opportunities for left-turning maneuvers to be made and consequently these vehicles and nonturning vehicles queued behind them will suffer longer delays before clearing the intersection.

As drivers suffer longer delays at an intersection, they may become impatient and tend to make maneuvers that are hazardous. Drivers delayed in turning left may suddenly make their turn in front of oncoming traffic that is so close that drivers of these opposing vehicles are forced to slow down or even brake quickly to a stop. If such an opposing vehicle is being followed too closely by another vehicle, a rear-end collision may result. On the other hand, the driver of a nonturning vehicle delayed for a long time behind a left-turning
vehicle may also become impatient and attempt a dangerous lane change or "passing-on-the-right" maneuver.

In order to increase the capacity and to diminish the hazard at a given location, a decision must be made to, a) either prohibit left turns or, b) to redesign the intersection. Prohibiting turns is not always the best solution as it may only transfer the problem elsewhere in the street system. In redesigning the intersection, the traffic engineer may either use multiphase signal control with existing intersection geometrics, a separate left-turn lane with existing two-phase signal control, or a combination of both multiphase control and channelized left turns.

The traffic engineer has volume warrants (30) (number refers to bibliography listing) to help him decide when a pretimed traffic signal should be placed at an intersection; but must rely on qualitative warrants or rules-of-thumb to decide on left-turn channelization. The need for quantitative or factual warrants to replace rules-of-thumb has stimulated the research described in this report.

Warrants for Left-turn Channelization.

Among the warrants already available to the traffic engineer are those contained in the Highway Research Board's Special Report No. 74 (1) and the A.A.S.H.O. "Policy on Geometric Design of Rural Highways" (2). These publications list several general purposes or "qualitative Warrants" for the introduction of channelization at intersections such as the separation of conflicts, reduction in excessive pavement areas, control of angle of conflict, etc.
In 1963 Failmezger (3) of the Oregon State Highway Department reported the development of a "relative warrant" for left-turn refuge construction. This warrant depends on an "index-of-hazard" which considers not only the volume of left turns and opposing traffic but also the pavement width, sight distances and speeds involved at specific locations. The Highway Capacity Manual (4) also contains warrants in terms of the effect of left turns on capacities of signalized intersection approaches.

The warrants derived from the hereinafter reported research are quantitative or "factual" in nature. They are related to traffic operational characteristics of an intersection including traffic volume, percent left turns, signal timing and vehicle delay. The number of vehicles delayed more than one signal cycle was the principal "figure-of-merit" used in this research.

**Methods for Determining Vehicle Delay.**

The three general methods for determining vehicle delay at an intersection are actual field studies, theoretical analyses and computer simulation.

The computer simulation procedure was selected for this study since it would be too costly and time consuming to perform an adequate number of field studies and theoretical analyses would not adequately represent the complex operational nature of a signalized intersection. Furthermore the researcher has precise control over the variables involved and can simulate hours of traffic operation in minutes.
Digital computer simulation is a relatively new operations research technique and has been applied to the traffic engineering field only in the last decade. Freeway traffic simulations were reported by Gerlough (5) in 1956, Perchonek and Levy (6), and Wohl (7). Intersection simulations previously reported have included those of Goode, Pollmar and Wright (8), Benhard (9), Kell (10, 11), Lewis (12), Gerlough and Wagner (13), and Bleyl (14). However, none of these studies addressed themselves to the effects of left turns.

Intersection Operational Characteristics.

In developing a simulation model of an intersection, the following elements of an intersection were considered most important:

1. The time distribution of vehicles arriving at the intersection.
2. Vehicle approach speeds.
3. Lane change movements.
4. Acceleration and deceleration characteristics near the intersection.
5. Car-following considerations.
6. Driver response to the yellow phase of a traffic signal.
7. Starting headways for vehicles in a stopped queue.
8. Gap acceptance requirements for left turning vehicles.

A detailed literature search revealed that, excepting the third and eighth ones, most of these characteristics could be adequately defined from previous research. Other research dealing with gap acceptance distributions has dealt either with vehicles entering or crossing a major street from a stop sign controlled cross street or left-turn gap acceptance on other than 4-lane streets.

Studies of the first type by Raff (15) and Bissell (16) were reported prior to this research and studies by Solberg and Oppenlander (17) and Wagner (18) were reported in 1965. Three student theses at
Yale by Kaiser (19), Clark (20), and Noblitt (21) provided left-turn acceptance data at uncontrolled intersections. Kell (22) provided acceptance data for left turns on two-lane, two-way streets.

Therefore the writer decided that it was a necessary part of this research to conduct a sufficient number of field studies to define left-turn gap acceptance characteristics for multilane approaches and, if possible, also define lane change relationships.

Objectives and Scope of the Research.

Objectives. The principal objectives of the research described in this report were as follows:

1. From field studies, the research was to provide a determination of a basic traffic characteristic—the driver's acceptable gap in the opposing traffic stream(s) for a left-turn maneuver—and its variation with individual drivers and traffic conditions of an intersection on a 4-lane street.

2. Through a practical application of digital computer simulation techniques, this research was to provide the traffic engineer with the following useful information:
   a. Factual warrants, based on delay criteria, for the introduction of separate left-turning lanes on the approaches to at-grade intersections on 4-lane streets.
   b. Design data for determining the required length of a separate turning lane, depending on operating traffic volumes in the intersection.
   c. Factual warrants, based on delay criteria, for the provision of a separate signal phase for a channelized left turn.

Scope of the Research. There are numerous types of intersections that could be included in a study of this kind. There are intersections on two-lane vs. those on multilane streets; there are intersections that have signalized traffic control vs. those with only stop

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sign or yield sign control or no control. There are three types of signalization—fixed time vs. semiactuated vs. full actuated; and there are isolated intersections in fringe or outlying urban areas vs. those in a progression system in a downtown area with pedestrian conflicts.

Due to the financial and time limitations of this research, it was not considered practical to conduct the great number of field studies and simulations that would be necessary to cover this intersection type variable. Therefore, the studies described in this report have been limited to isolated 4-lane major-arterial signalized (fixed-time) intersections which are considered to be the most critical type in most cities. Four geometric and signal variations permitted were:

1. No left-turn channelization, no separate turning phase.
2. Left-turn channelization, no separate turning phase.
3. Left-turn channelization and separate turning phase.
4. Left-turn channelization and unsignalized.

For the above described intersections, the research was concerned with the following:

1. Conduct of 6 principal field studies to determine the acceptable gap distribution(s) applicable to drivers of vehicles turning left from the inside lane of one approach across the two opposing lanes of traffic.
2. Development of a simulation model to represent the operation of traffic at such intersections, utilizing the gap acceptance characteristic determined in field studies along

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1 This intersection type was not studied in the field or simulated but it was considered the next step in intersection design, when variation No. 2 fails to provide the needed capacity.

2 This type of intersection was studied in the field, since left-turn characteristics were comparable to that of a signalized intersection when the signal display is green.
with other commonly accepted intersection operational characteristics.

3. Performance of numerous simulation runs to determine vehicle delay due to the presence of left turns and likely queue accumulations under each set of conditions.
   a. Volume levels from free-flowing to capacity conditions.
   b. Percentage of left turns from 0 to 30 percent of total inside lane volume.
   c. Signal cycle lengths from 50 to 90 seconds.
   d. Signal cycle splits from 40 - 60 to 60 - 40.

4. Performance of additional field studies to measure the number of vehicles delayed more than one signal cycle length at typical intersections for verification of the simulation model's ability to predict vehicle delay.

5. Development of relationships relating delay and queue length with volume and percent left turns that can be used to determine when left turns should be separated, when a separate signal phase should be provided for a channelized left turn, and how long such channels should be made under each set of conditions.
INTERSECTION FIELD STUDIES

Variables Studied.

In order to evaluate gap acceptance of left-turning drivers and to provide other data for the purpose of verifying simulation output it was decided that the following variables had to be measured in any field studies conducted:

1. Arrival time of each left-turn vehicle at intersection.
2. Arrival time of each opposing vehicle in the intersection center.
3. Waiting position of left-turn vehicle.
4. Time of actual turn maneuver and signal phase at that time.
5. Type of gap accepted or rejected.
7. Sex and approximate age of driver.
8. Arrival and departure time of each vehicle from intersection approach "system."
9. Traffic volumes operating in both directions.
10. Percentage of turns in each lane.
11. Signal cycle length and phasing split.
12. Length of study period.

Site Selection.

It is difficult to locate very many intersections with all the desirable characteristics one would prefer for study purposes. The intersections finally selected were not all ideal but did provide a good range of traffic volume and percentage of turns.

Each of six of the largest cities in Texas was visited and 78 intersections were inspected as possible study sites. On the basis of preliminary investigations, six intersections were selected for detailed studies. Only four of the six locations studied provided useable data; these were located as follows:

1. Austin, 24th Street at Lamar Boulevard.
2. Houston, Shepherd Drive at West Gray Street.
3. Houston, Franklin Street at Caroline Street.
4. Waco, North 18th Street at Waco Drive.

Study Methods Considered and Selected.

Accepted methods for obtaining the required data previously outlined include the use of a multiple event recorder connected to road tube actuated air switches and hand switches, 16 mm motion-picture photography, 16 mm time-lapse photography, or combinations of these methods. Each of these techniques was used for at least two hours of study at one location.

The intersection of 24th Street and Lamar in Austin was studied by motion picture techniques alone utilizing a special relay connected to the signal controller which only permitted pictures to be taken while the green phase was displayed for 24th Street. The Waco studies were primarily 20-pen recorder studies but some overlap with 16mm time-lapse photography was provided (2 hours on each approach).

On the basis of these studies it was decided that the most satisfactory and economical procedure, for both field study and data analysis time standpoints, was the time-lapse photography technique. Therefore, this procedure was used for both Houston studies.

Although principally interested in 4-way signalized intersections, it should be noted that one of the locations studied was an unsignalized tee intersection (Franklin at Caroline in Houston) with relatively high volumes. This was permitted for the following reasons:

1. Gap acceptance for left turns from a stopped position at an unsignalized intersection should be the same as that for operation on the green phase of a signalized intersection.
2. A relatively high percentage of left turns at this type of intersection are made without stopping and therefore gap
sizes required for this different maneuver could be evaluated and applied to the comparable maneuver at a signalized intersection.

**Time-Lapse Photography Method.**

A Bolex 16 mm camera and tripod was set up on either a roof-top or a platform truck within 250' of the intersection being studied. The single frame button on the camera was actuated at 1 second intervals through a special electrical timer-solenoid circuit. A typical setup for this type of study is shown in Figure 1 (also shown is a view of the accompanying 20-pen recorder setup at the particular intersection shown). A clock or watch was included in the field of view to establish time of day and provide a check on the frame interval obtained.

This technique required only one-tenth as much film as the motion picture technique and still provided gap measurements to an accuracy of ± 0.5 second. A filming technique is more desirable than the 20-pen

![Figure 1. Time Lapse Camera Equipment Set-Up—18th St. at Waco Drive.](image-url)
recorder method, since a complete picture of the intersection system is available at any instant and complete analysis of a situation can be obtained by running and rerunning the film through a projector.

Typical film strips obtained by this method are shown in Figure 2. All such films were analyzed through use of the Bell and Howell Time-and-Motion projector. For each left turn a data summary was provided as follows:

1. Beginning of the traffic signal green phase.
2. The length of the signal cycle.
3. The type of left-turn vehicle: automobile, single unit truck, etc.
4. Whether the left-turn vehicle was stopped, creeping, or moving as the turn maneuver was initiated.
5. The position from which the turn was made: stop line, center of intersection, or half-way into intersection.
6. The arrival time, looking-for-gap time, move-out time and turn completion time for the left-turn vehicle (recorded as frame number).
7. Approach and opposing volume levels by lane and number of turns.
8. Opposing vehicle speeds; indication of slowing due to turn vehicle.
9. The vehicle type (also typed by maneuver) and arrival time at the center of the intersection for every opposing vehicle this turn vehicle encounters.

All this information was transferred to punch cards with items 1 to 8 on one card and item 9 placed on two additional cards, one for each opposing lane of traffic.

A Fortran program was written for an IBM 709 digital computer to analyze these data. The information provided by this analysis included the following:

1. Identification by green phase number and left-turn vehicle number.
2. Left-turn vehicle type, turn type and starting position.
3. Signal phase that turn was initiated on.
4. Queue of vehicles behind left turn.
5. Approach and opposing volume levels per signal cycle.
Figure 2a. Typical Time-Lapse Film Strips—18th at Waco Dr.
(1) Turning on Amber, Yielding to Opposing Right Turn

(2) Gap Acceptance

Figure 2b. Typical Time Lapse Film Strips—Houston, Texas.
6. Vehicle type and speed for opposing vehicle in front of which left turn was made.
7. A listing of all rejected gaps by type and size.
8. The accepted gap by type and size.
9. An indication as to whether or not the turn was forced, i.e. caused opposing vehicles to slow or stop.
10. Delay to left-turn vehicle (move-out time less arrival time).

There were four types of gaps considered in this analysis of 4-lane street studies, as shown by Figure 3. There are two types of lane gaps, i.e. gaps formed by two successive inside lane vehicles or two successive outside lane vehicles. In addition there are two types of offset gaps, i.e. gaps formed by a vehicle in one lane trailed by a vehicle in the adjacent lane.

Other studies (17, 18, 19) have distinguished between "gaps" and "lags" in developing acceptance distributions. The difference in these two terms is illustrated in Figure 2b (2). Note that the first left-turn vehicle accepts a "gap" formed by the bus and the trailing station wagon. The second left-turn vehicle accepts a "lag" or the time elapsed between its arrival at the center of the intersection and the arrival of the station wagon at the same location (in its own lane).

Field Study Results.

The principal objective of the field studies previously described was to determine the distribution(s) of acceptable gaps for left-turning vehicles on four-lane arterial streets. This was accomplished as hereinafter described including variations for moving and stopped vehicles and for different types of opposing gaps.

Other characteristics of left turns were also noted such as the waiting or starting position in the intersection area, the percentage
FIGURE 3. OPPOSING TRAFFIC STREAM GAP TYPES CONFRONTING LEFT TURN VEHICLES.
of left turning vehicles that "jump-the-gun" at the start of the green phase, the frequency of left turns clearing the intersection after the end of the green phase, the percentage of left-turn drivers accepting gaps smaller than the previously largest gap rejected, and the variation of gap acceptance with driver characteristics.

1. Turn Types and Starting Positions.

Aside from left turns that "jump-the-gun," which are discussed a little later, most left turns at a signalized intersection under moderate to heavy traffic conditions will be made from a stopped or waiting position in the intersection proper.

Other left turns may be made while the vehicles are slowly moving (or creeping) into the intersection proper, while still others are made by vehicles that never slow much below maximum permissible turn speed. Under relatively heavy traffic conditions these vehicles are those that follow on the "heels" of other left-turn vehicles; under relatively light traffic conditions, the majority of the turns are likely to be nonstopping turns.

Analysis of data for 174 turns at the most typical intersection studied, W. Gray at Shepherd in Houston, showed 126 or 72.4 percent of these turns were made from a stopped position. Of these 126 turns 75 or 59.6 percent were waiting in a position near the center of the intersection, 41 or 32.5 percent were waiting in a position about halfway into the intersection from the stop line, and the remaining 10 turns (9 of whom "jumped-the-gun") were made from the stop line.

At the unsignalized intersection of Franklin and Caroline streets in Houston, data for 690 left turns showed 447 or 64.9 percent were moving type turns.
2. Left Turn Gap Acceptance.

As illustrated by Figure 3, there are four possible gap types to be found in the opposing traffic stream. In each of the three principal studies, gap acceptance distributions were determined for each gap type. For each particular gap size (nearest 0.5 sec.) evaluated, the percent of gaps accepted is defined by the number of these gaps accepted divided by the total number of these gaps accepted and rejected. Turns that "jumped-the-gun," forced their turn, or completed their turn after all opposing traffic had stopped were not considered to have accepted a normal gap.

a. Turns From a Stopped Position. The largest sample of turns from a stopped position came from the study of the intersection of Franklin and Caroline Streets in Houston. A total of 232 accepted gaps were distributed as to type as follows: 51 Type 1, 66 Type 2, 55 Type 3, and 60 Type 4 gaps. These 232 turns also rejected a total of 809 gaps. The resulting cumulative distribution of acceptable gaps by type is shown in Figure 4. (Note the considerable scatter of data points.)

When unsatisfactory turns (forced, unopposed, etc.) were eliminated from the 126-vehicle sample at the West Gray and Shepherd location there were only 76 turns left. Accepted gaps distributed as to type included 14 Type 1, 21 Type 2, 11 Type 3, and 30 Type 4 gaps. These sample sizes were too small to provide satisfactory cumulative curves but there did appear to be a distinct difference between Type 4 gaps and the other 3 combined as shown in Figure 5.

At the Waco location, only those turns that could be verified by time-lapse films in the same manner as the other two studies were used
for further comparison. With a high percentage of turns being made without opposition or "jumping-the-gun," only 35 of 232 turns were able to be analyzed and this provided data on 16 Type 4 acceptable gaps as compared with 19 gaps of Types 1, 2, and 3. The cumulative distributions for these turns are also shown on Figure 5.

b. Turns From a Moving Position. The gap accepted by a moving left-turn vehicle is not defined by two opposing vehicles but by the difference in arrival times (at the intersection center) of the turning vehicle and the nearest opposing vehicle. (Some authors designate such a gap as a lag.) Therefore there were only two types of gaps to be evaluated; one with the nearest opposing vehicle in the outside lane (Type 2) and the other with the nearest opposing vehicle in the inside lane (Type 1).

The 447 moving left turns filmed at Franklin and Caroline Streets in Houston were classified as either creeping or moving at normal turn speed as they began their turn. An analysis of gap acceptance distributions for both conditions showed no significant difference between the two for both Type 1 and Type 2 gaps. The data were then separated into two groups by gap type providing 185 Type 1 gaps and 262 Type 2 gaps.

Since a moving turn does not reject a gap, it was necessary to check the first gap rejected by each of the 243 left turns that stopped at this location before making their turn. This analysis provided 129 Type 1 gaps and 114 Type 2 gaps that were rejected. The cumulative gap acceptance distributions developed from these analyses provided the two curves shown on Figure 6. As would be expected for a given gap size, a lower percentage of drivers will accept the gap, if it is formed by an opposing vehicle in the outside lane, as compared with one in the
FIGURE 4. PROBABILITY OF LEFT TURN VEHICLE ACCEPTING GAP FROM STOPPED POSITION AT UNSIGNALIZED, CHANNELIZED TEE INTERSECTION; FRANKLIN AT CAROLINE, HOUSTON.
FIGURE 5. PROBABILITY OF LEFT TURN VEHICLE ACCEPTING GAP FROM STOPPED POSITION AT SIGNALIZED 4-WAY INTERSECTIONS.
inside lane; it takes longer to clear the outside lane than the inside lane when making the left turn.

c. Left Turns That "Jump-The-Gun." It was observed that many drivers whose left-turn vehicle is at the head of the inside lane or left-turn channelization queue when the signal changes to a green indication will "jump-the-gun," i.e. make their turn before the opposing traffic enters the intersection. In some states this maneuver is outlawed, in others it is encouraged; but it definitely does occur at the intersections studied in several Texas cities.

To determine the probability of its occurrence the number of signal cycles at which a left turn was observed at the head of the inside lane or channelized queue was divided into the number of cycles when such a vehicle "jumped-the-gun." The results for 5 signalized approaches are shown in Table 1. It is interesting to note that the same percentage occurred for the total observations of each type of intersection.

<table>
<thead>
<tr>
<th>Intersection Location</th>
<th>Approach Type</th>
<th>No. of Signal Cycles</th>
<th>With L.T. at &quot;Jumping-the-Gun&quot;</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>With L.T. at Head of Queue</td>
<td></td>
</tr>
<tr>
<td>Houston</td>
<td>nonchannelized</td>
<td>130</td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td>Austin #1</td>
<td></td>
<td>65</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Austin #2</td>
<td></td>
<td>65</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>260</td>
<td>55</td>
<td>8</td>
</tr>
<tr>
<td>Waco #1</td>
<td>Channelized</td>
<td>127</td>
<td>79</td>
<td>8</td>
</tr>
<tr>
<td>Waco #2</td>
<td></td>
<td>127</td>
<td>86</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>254</td>
<td>165</td>
<td>24</td>
</tr>
</tbody>
</table>
Figure 6. Probability of left turn vehicle accepting gap without stopping at unsignalized, channelized tee intersection; Franklin at Carolina, Houston.
d. Change of Required Acceptable Gap. As hypothesized there were a number of drivers of left-turn vehicles who accepted a gap that was smaller than the largest one previously rejected. In general the difference was only about 0.5 to 1.0 second.

The frequency of such a change in gap requirements and consequent probability of occurrence is shown in Table 2. The writer attempted to relate this occurrence with vehicle waiting time and opposing traffic stream density but no relationship was apparent.

<table>
<thead>
<tr>
<th>Study Location and Approach Type</th>
<th>No. of Left Turns</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Accepting Gap Smaller Than Largest Rejected</td>
</tr>
<tr>
<td>Houston—Nonchannelized</td>
<td>119</td>
<td>14</td>
</tr>
<tr>
<td>Waco—Channelized</td>
<td>241</td>
<td>22</td>
</tr>
<tr>
<td>Grand Total</td>
<td>360</td>
<td>36</td>
</tr>
</tbody>
</table>

e. Turns Made on Yellow and Red Phases. Many left turns were completed after the signal phase had changed to yellow or even to red. Observation of 260 cycles of operation on 3 nonchannelized approaches in Houston and Austin found 58 left-turn vehicles still waiting to turn when the green phase ended. All 58 of these turns were then completed, 40 percent of them on the red phase.

At these and other locations, it was further noted that more than one vehicle may turn after the end of the green phase. As many as four
turned on one occasion at the Waco location, the last one almost colliding with cross street vehicles that had started up legally (see Figure 2a (2)).


The first study conducted during this research utilized the 20-pen recorder and one observer coded in the type of left-turn vehicle, the sex of the driver and the approximate age group of the driver.

Without regard to gap type, there were 512 left turns whose average acceptable gaps were tabulated as shown in Table 3. There is no significant difference in the grand total means in view of the fact that the measurement of gap size was accurate only to the nearest 0.5 second. There were too few trucks to provide a meaningful comparison with passenger cars.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Male Drivers</th>
<th>Female Drivers</th>
<th>Grand Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. Range</td>
<td>No.</td>
<td>Mean Gap</td>
</tr>
<tr>
<td>1</td>
<td>&lt;25</td>
<td>29</td>
<td>6.64 sec.</td>
</tr>
<tr>
<td>2</td>
<td>25-45</td>
<td>260</td>
<td>7.01</td>
</tr>
<tr>
<td>3</td>
<td>45-65</td>
<td>60</td>
<td>6.92</td>
</tr>
<tr>
<td>Grand Total</td>
<td>349</td>
<td>163</td>
<td>7.10</td>
</tr>
</tbody>
</table>

TABLE 3

GAP ACCEPTANCE BY DRIVER CHARACTERISTICS
18TH ST. AT WACO DR., WACO, TEXAS

24
On the basis of these results it was decided not to complicate further studies with the classification of driver characteristics. This was further justified in that only the overall gap acceptance distribution was necessary for simulation purposes; the computer does not need to know the age or sex of each driver.

Comparisons With Other Research.

It is of interest to note how the distributions developed in this study compare with those of other researchers. Figure 7 compares two distributions with combined (all gap types) data from this study with the combined left turn gap acceptance data from the recent validation study of Traffic Research Corporation (23) and the left-turn gap acceptance from a cross street developed by Bissell (16) and Solberg and Oppenlander (17).

The gap acceptance distributions obtained from the intersection of Franklin and Caroline compare quite favorably with those obtained by Traffic Research Corporation. For turns from a stopped position the distribution is almost identical with that obtained for the combined California data. The moving turn gap distribution is similar to the TRC acceptable lag distribution but has a steeper slope.

As would be expected, there is a definite difference between distributions obtained for left turns from cross streets under stop sign control and the left turns across opposing traffic streams as reported in this report. Another comparison with Kell's data (22) in Figure 15 shows that left turns across a single lane opposing stream require smaller gaps than turns across a two-lane stream.
FIGURE 7. GAP ACCEPTANCE COMPARISON.
Statistical Analyses of Gap Acceptance Distributions.

The probit method of analysis (24) and Chi-Square test were used to determine differences in the distributions shown in Figures 4, 5, and 6. These analyses showed significant differences between acceptable gap distributions for:

1. Type 4 gaps vs. all other types for turns from a stopped position.
2. Moving turns vs. turns from a stopped position.
3. Type 1 vs. Type 2 for moving turns.

Summary of Field Study Findings.

On the basis of the research reported herein the following seem to be the most important findings of the field studies conducted. Although it should be noted that these are based on a limited number of studies, it is felt that the results are representative of similar types of signalized intersections.

1. A time-lapse 16 mm filming technique that utilizes one second intervals was found to be the most satisfactory and economical method for studying gap acceptance characteristics of left turns at a signalized intersection.

2. More than one gap acceptance distribution or one critical value is necessary to cover the existing range of left turn types on a 4-lane street.
   a. Moving turn gap or lag acceptance distributions are significantly different than distributions for turns made from a stopped position.
   b. Shorter gaps (lags) are required for most moving turns, if the next opposing vehicle is in the inside lane as compared with one in the outside lane.
   c. For turns made from a stopped position there is evidence that Type 4 gaps (formed by a leading outside lane vehicle and a trailing inside lane vehicle) will more likely be accepted than any of the other three gap types of the same size.

3. There was no appreciable difference between gap acceptance requirements for left turns and other turn characteristics on a
channelized approach as compared with those on an unchannelized approach. In general, it is very rare that a gap smaller than 2 seconds will be accepted or that one larger than 8 seconds will be rejected.

4. With the exception of turns that "jump-the-gun" or turn without stopping almost every left turn is made from a waiting position within the intersection area ahead of the stop line. There are likely to be twice as many turns made from a position (front of vehicle) at the center of the intersection than from a position about half-way between the stop line and the intersection center.

5. There are a significant number of drivers who "jump-the-gun" in turning left at the beginning of a green phase and this feature should be incorporated in any left-turn model. The probability of this occurring appears to be about 0.15 for all left-turn vehicles at the head of a queue when the signal first displays the green phase.

6. Although 10 percent of left-turning drivers accepted gaps smaller than the largest gap rejected, this research did not establish any significant relationships with driver delay or opposing traffic stream characteristics to explain this phenomenon. The difference in gap sizes being generally less than 1 second makes this result of little value to a simulation model.

7. A large number of left turns will be completed after all opposing traffic has passed and many will even turn after the green phase has ended. This fact makes it difficult to collect gap acceptance data under light to moderate traffic conditions.
Traffic Operational Characteristics Considered.

The operation of traffic on an intersection approach or within the intersection area consists of several vehicle maneuvers, vehicle interactions and driver responses. The following discussion, related to Figure 8, illustrates these characteristics.

As vehicles approach within several hundred feet of an isolated intersection, they are not considered to be operating under the influence of the intersection. For this condition, the arrival of vehicles at a point would be generally random and the distribution of successive time spacings (headways) between vehicles would conform to an exponential relationship.

At some point on the approach, point (1)\(^1\) in Figure 8, drivers will begin to alter their speed due to the presence of slowing vehicles ahead or the build-up of a queue of stopped vehicles at the intersection. Actually the point (2) at which any given vehicle begins to slow will depend on the conditions existing at that time. A single independent vehicle approaching on the red phase may not begin to slow until half way to the stop line from point (1); whereas the trailing vehicle of a platoon of approaching vehicles may begin to slow soon after reaching point (1) due to vehicle interaction.

Due to a stop condition ahead, the driver at point (2) realizes the necessity for stopping and will likely begin to decelerate at a low rate in zone (3). Zone (3) is also the general area where

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\(^1\)Note: All numbers in parentheses on pp. 29 and 31 refer to numbers in Figure 8.
FIGURE 8. ILLUSTRATION OF PRINCIPAL TRAFFIC OPERATIONAL CHARACTERISTICS AT A TYPICAL SIGNALIZED INTERSECTION.
approaching drivers may be confronted suddenly with a yellow indication on the signal head and must decide whether to initiate deceleration or to continue through the intersection proper, (7), this latter maneuver being completed during the end of the yellow phase or many times during the beginning of the red phase.

At any point on the approach between point (1) and the intersection proper, (7) there can be interaction between successive vehicles in a traffic lane. Generally this interaction results when a faster moving vehicle overtakes a slower moving vehicle and the driver of the faster vehicle adjusts his speed according to the speed differential and headway between the two vehicles. If the driver of the faster vehicle intends to turn at the intersection and he is already in the proper lane for his turn, then he will most likely remain in this lane; even though traveling slower than desired at the moment, he will eventually have to slow to a turning speed anyway. However, the driver of a nonturning vehicle in the same situation may attempt to improve his position on the approach by passing the slower vehicle through a lane change maneuver.

As a left-turning vehicle nears the intersection proper in the inside lane with no vehicles immediately in front of it, its driver begins to evaluate the opposing traffic stream(s). Generally, each driver requires a certain size gap in the opposing traffic, (6) in Figure 8, before he is willing to initiate his turning maneuver. He may find such a gap before he has to stop his vehicle in the intersection and will be able to make his turn immediately. However, under near capacity conditions he will have to stop somewhere near
point (4) and wait for a suitable gap before making his turn, many times having to turn after the yellow phase has begun and all opposing traffic has stopped. If the left-turner has been delayed for a long time, he may become impatient and accept a shorter gap than he originally desired.

If the next inside lane vehicle is not a left-turner, he may not be content to wait for the turning vehicle to move and will evaluate the adjacent outside lane for a gap large enough to permit him to change lanes, as at point (5) in Figure 8. Actually almost any of the nonturning, inside lane vehicles queued up behind the waiting left-turn vehicle may decide to make this change maneuver. In addition, other drivers make lane changes while still approaching the stopped queues in hopes of improving their position and minimizing their delay in getting through the intersection.

With the signal head displaying the red phase, stopped queues of waiting vehicles build up on the approach to an intersection. When the signal display changes to green, there is usually a short starting delay before the two queues of vehicles begin to move into the intersection proper. This is the time required by the first driver to perceive the light change, react to it and initiate his vehicle's acceleration from its stopped position and enter the intersection proper. (Occasionally, the first vehicle in the inside lane may be a left-turning vehicle whose driver is primed to move the instant that the light turns green; he will "jump-the-gun" and make his turn before the opposing lead vehicles reach the center of the intersection.) Succeeding vehicles in a queue proceeding across the intersection will follow with progressively smaller headways
between successive vehicle entries until some relatively constant value is reached for the last few vehicles.

From the preceding discussion, one should realize the complexity of intersection operation and why computer simulation represents the only practical way to study it. However, in developing a simulation model, one must be careful lest he attempt to build a perfect representation of the real intersection. Not only is such perfection difficult and time consuming to obtain but the resulting model will likely be very inefficient, uneconomical, and impractical to run even on the fastest computers. Therefore one should probably best strive to describe a model which will adequately represent the most important operating characteristics and ignore the unusual or insignificant events. The simulation model described in this report was developed with this basic philosophy in mind.

The Basic Form of the Model.

The simulation model used in this research represents the traffic operation at an isolated 4-lane major arterial signalized (fixed-time) intersection.

Only the traffic on the 4-lane major arterial was simulated; the presence of 4-lane, 2-way cross street traffic was acknowledged by the percentage of the signal cycle not available to the major arterial. Both directions of traffic on the arterial were represented for a distance of 1,000 feet back from each approach stop line. As previously discussed, such a length was considered as the practical limit of the intersection's influence on approaching traffic. In addition, this length is necessary to provide storage room when demand volumes exceed capacity.
The principal geometrics of the intersection system represented are shown in Figure 9. Key points are located by a coordinate (or length of travel path) distance from the point of entry into the intersection system. Likely paths followed by turning vehicles are shown in Figure 9 and these paths determined the corresponding exit coordinates for these vehicles. Once a vehicle had reached or crossed any exit coordinate, it was considered out of the system and no longer had any effect on other vehicles still in the system.

The basic method of simulation used was the periodic scan technique employing Monte-Carlo procedures. The system was scanned every second of real time and every vehicle in the system was processed according to its desires or as controlled by operating conditions in the system.

The time interval of one second was selected as being the optimum value considering several factors. The most important of these is the minimum intervehicular headway—vehicles cannot enter the intersection system in closer than one second intervals. To require scanning at intervals smaller than one second would increase the precision of the arrival distribution but would also double or quadruple the time and cost of simulation. Furthermore, one second represents a good average figure for driver perception and reaction time in most driving situations. The interval does not need to be any smaller than the smallest event simulated and yet not be too large as to miss some significant events.

Input parameters of volume levels, percent turns, signal cycle length, and percent red signal time were read into the computer before each run. The output of each run for each lane of traffic included
FIGURE 9. PRINCIPAL GEOMETRICS AND VEHICLE PATHS FOR SIMULATED INTERSECTION.
vehicle travel time, number of vehicles delayed more than one signal cycle length, and the traffic volume clearing the intersection. Additional output displayed actual arrival gap and desired speed distributions as well as other intermediate values helpful in determining whether or not a valid simulation was obtained.

Operational Characteristics Selected.

The following discussion briefly summarizes the basic traffic characteristics selected and represented in the simulation model. The sources of these data are indicated by the names and numbers in parentheses (referring to the bibliography).

1. **Input Distribution.** Vehicles entered the intersection system in each lane according to time spacings determined from the offset negative exponential distribution [Gerlough (25)] with minimum permissible headways of one second.

2. **Vehicle Type.** During any simulation period it was assumed that there was a constant probability of any new arrival being a turn vehicle. This value varied with the percent turns specified.

3. **Vehicle Speed.** A typical cumulative speed distribution with a mean speed of 28 mph was selected to represent the range of desired vehicle speeds at the point of entry to the intersection system.

4. **Decelerations.** An independent or lead vehicle approaching the intersection with a signal indication of red was initially decelerated at no more than 3.5 ft/sec$^2$ to simulate observed practice. When vehicles were decelerating to a stop in response to the yellow phase display or slowing behind much slower vehicles, deceleration rates approaching 12 ft/sec$^2$ as a maximum were permitted.
5. **Car Following Relationship.** The generalized equation suggested by Drew (26) was used to control the spacing of all vehicles following another at a speed greater than that of the preceding vehicle. It also was used for decelerating a lead vehicle to a stop by introducing a pseudo vehicle whose speed was zero and whose position was 25 feet ahead of the desired stop position. Based on personal observations at numerous intersections, the writer feels that the stopped queue intervehicular spacing of 25 feet is realistic. For the conditions represented in the model, Drew's equation becomes:

$$\ddot{x_i}(t + T) = 1980 \left[ \frac{\dot{x}_i - 1(t) - \dot{x}_i(t)}{[x_i - 1(t) - x_i(t)]^2} \right]$$  \hspace{1cm} (1)

6. **Response to Yellow Phase Signal.** Whenever the signal first displays the yellow phase, the first vehicle behind the stop line in each lane was checked for the likelihood that it would stop or continue through the intersection. If this vehicle was to continue, then the next vehicle would be checked; the stopping of subsequent vehicles was guaranteed by the car following relationship. The curves developed by Olson and Rothery (27) shown in Figure 10 were approximated by straight lines and used to determine this probability for each vehicle in question through random selection procedures.

7. **Starting Headways.** Through and right turn vehicles entered the intersection proper from a stopped queue with successive headways as described by Greenshields et al. (28). The basic spacing relationship used by Lewis (12) provided these headways satisfactorily with the lead vehicle accelerating at 3 ft/sec$^2$ and trailing vehicles having 1 second reaction times. This relationship is shown in equation 2.
FIGURE 10. REPRESENTATIVE CURVES FOR THE PROBABILITY OF STOPPING FOR THREE SPEED ZONES. (THE DOTTED CURVES ARE THOSE OF THE WEBSTER DATA.) AFTER OLSON AND ROTHERY (30).
\[ z_a = \frac{1}{2} [V_{t-1} + (V_{t-1} + \bar{A})] \]  

where

\( z_a \) = distance that vehicle travels in a one second interval
\( V_{t-1} \) = actual vehicle speed one second before time \( t \)
\( A \) = acceleration rate permitted

When accelerating from a stop or slowed speed, a vehicle was permitted to regain its desired speed unless restricted by the preceding vehicle.

8. **Left Turn Characteristics.** Left turn vehicles were operated according to the relationships developed by this research and summarized on previous pages (pp. 27 and 28).

9. **Lane Changes.** It was previously mentioned that a great many drivers operate their vehicles on intersection approaches to minimize their delay in clearing the intersection. This generally involves the performance of lane changes to pass a slower vehicle or a stopped vehicle or queue waiting for an opportunity to turn left. However, the writer is unaware of any studies that have been conducted to specifically evaluate this characteristic; and his limited investigations did not produce any relationship to account for the random behavior observed.

There does appear, however, to be a relationship between the lane distribution of traffic volume entering the intersection and the percent of left turns. Table 4 shows a generalized form of this relationship. It was assumed that the use of this table would account for any lane changes that would occur in the real situation.
10. **Rules of Operation.** All of the preceding characteristic descriptions might also be considered as "rules of operation" for the simulation model. Additional rules include:

1. It was assumed that there were no accidents or vehicular breakdowns in the intersection.

2. The position of each vehicle in the system was referenced to its front bumper.

3. There were no pedestrian interferences in the intersection.

4. No vehicles were permitted to enter or leave the intersection approaches between the point of entry and the intersection area—i.e. no driveways or minor cross streets.

5. All right turns were made only from the outside lane of an approach. Similarly, all left turns were made only from the inside lane of an approach.

6. A vehicle turning left from the intersection center entered the outside lane of the cross street and yielded the right-of-way to a right-turning vehicle from the opposite approach.

7. Commercial vehicles are not specifically provided for in this model. Use of an equivalent number of automobiles should be used to account for trucks operating in the real intersection.

**Simulation Program Logic.**

After the simulation model was defined as to type and characteristics, the next step in its development was concerned with computer
program logic. This step began with the development of flow charts which outlined how all the desired operational characteristics were to be tied together into a working model of the unchannelized intersection. As is summarized later, slight modifications were made to apply the model to channelized intersections. Figure 11 represents the overall flow chart for the simulation program. Basically there was one main program with 6 sub-programs.

Input Parameters and Production Runs.

When the complete program was considered to be operational, production runs were made. For any set of runs it was necessary to specify several input parameters as listed below:

1. The desired volume level for each approach.
2. The desired percentage of turns for each lane.
3. The signal cycle length in seconds.
4. The signal cycle "split" or percent of the time that the signal indication will be red.

Each run was for one hour of real time and each set of runs was preceded by a "warm up" period of one hour. This period permitted the system to become loaded with normal traffic before the desired production runs were made (or sampled).

Simulation Model Efficiency and Validity.

The simulation model previously described is relatively efficient in simulating the operation of traffic through a signalized intersection. A basic set of 6 one-hour runs (6 different volume levels) is executed in 18 minutes of IBM 7040 computer time. This is equivalent to a ratio of about 20 to 1 of real time to computer time.
INITIALIZATION

READ INPUT PARAMETERS FOR NEXT RUN

RESET NECESSARY ARRAYS

INCREASE SIGNAL TIMER BY ONE AND DETERMINE PHASE DISPLAYED

SELECT LANE, DETERMINE LEAD VEHICLE

LT. TYPE OF VEHICLE

CALL LTLEAD

CALL THRULE

RT. THRU

CALL RTLEAD

ANY MORE VEHICLES IN LANE?

Yes

INCREASE LANE COUNT +1
DETERMINE DESIRED SPEED AND IF TURN VEHICLE

No

NEW ARRIVAL

Yes

HAS SIMULATION PERIOD ENDED?

No

PRINT SIMULATION RUN RESULTS SUMMARY
PUNCH DATA CARDS FOR REGRESSION ANALYSIS

STOP

Yes

LAST RUN?

No

FIGURE II. OVERALL FLOW CHART - MAIN PROGRAM.
The validity of the simulation model was checked by entering actual field data into the relationships developed from the simulation and comparing actual delay values with predicted delay values.
SIMULATION RESULTS

The principal findings of this research were derived from the traffic simulations previously described. Under the controlled conditions of the simulation model it was possible to relate traffic operational characteristics to vehicle delay. These relationships were used to suggest possible warrants for the separation of left-turning traffic at signalized intersections on 4-lane undivided arterial streets and to aid in the determination of the length of separate left-turn lanes.

Variation of Simulation Traffic Components.

A typical generated gap distribution for a traffic lane was compared with its corresponding theoretical offset negative exponential distribution and tested for "goodness-of-fit" by the Chi Square Test. It proved to be a good representation of the theoretical with probability value of 0.30. A similar comparison of a typical generated cumulative speed distribution with its desired input showed an even better fit.

The variability of the generated traffic volume and percent turns is shown by Table 5. This tabulation was used to determine the minimum sample size necessary to obtain delay data for warrant development.

TABLE 5
VARIATION OF INPUT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Sample Size</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Volume (Vehicles Per Hour)</td>
<td>625</td>
<td>20</td>
<td>630.4</td>
<td>21.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>625.3</td>
<td>21.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>624.4</td>
<td>21.27</td>
</tr>
<tr>
<td>% Left Turns</td>
<td>20%</td>
<td>20</td>
<td>20.19</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>20.19</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>20.33</td>
<td>1.90</td>
</tr>
</tbody>
</table>
Delay Criteria.

Although some regression analyses of simulation runs were made, the results did not provide a convenient tool for the designer's use. Instead it was felt that a set of probability curves that would show the chances of certain levels of delay occurring would be more desirable. This is in line with the level of service-design concept presented by Keese, Pinnell, and Drew (29) in a recent paper. As they put it, "these curves represent an attempt to put such a decision in the hands of highway administrators and designers."

It was decided to determine the probability of inside-lane vehicles being delayed more than one signal cycle length. To do this all of the independent variables were maintained at a constant input value for each of 20 simulated hours of traffic operation. A sample size of 20 was selected as being large enough to provide probability values to the nearest 0.05. It also was large enough to provide a sample with no greater degree of variability of input parameters than a somewhat larger sample as shown in Table 5. Such a sample size also greatly reduced the amount of computer time required to gather the data.

Separate Left-Turning Lane Warrant.

Figure 12 represents a basic set of probability curves suitable for use by the traffic engineer in deciding when to separate left-turning movements. The curves provide the probability of 3 different delay levels being exceeded. These levels are designated as follows:

<table>
<thead>
<tr>
<th>Designation</th>
<th>Delay Level Represented</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-no inside lane vehicles delayed more than one signal cycle length.</td>
</tr>
</tbody>
</table>
FIGURE 12. PROBABILITY OF DELAYS GREATER THAN ONE CYCLE LENGTH WHEN OPPOSING VOLUME = 500 V.P.H. (± 50), [CYCLE LENGTH = 70 SEC.; 50% GREEN TIME]
UNCHANNELIZED APPROACH

CHART I (5 - 15% LT)*

Probability of Delay

CHART II (15 - 25% LT)

5% Delayed

CHART III (25 - 35% LT)

60% of Time

*% Left Turns of Inside Lane Volume Only

Total Approach Volume - VPH (± 50)
5 -5 percent of all inside-lane vehicles delayed more than one signal cycle length.

10 -10 percent of all inside-lane vehicles delayed more than one signal cycle length.

To obtain these curves input parameters for generating opposing volume, signal cycle length, signal cycle split and percentages for turns were all held constant, while approach volume was varied for each set of 20 runs in increments of 100 vehicles. Because of the variability reported in Table 5, the charts are plotted for 100 vehicles per hour volume ranges and 10 percent left turning ranges; 95 percent of all simulated results fell within these ranges for each combination.

A set of 20 runs was obtained for each combination of the following parameter values:

1. Approach volume of 300, 400, 500, 600, 700, and 800 vph.
2. Opposing volume of 500, 625, and 750 vph.
3. Percent left turns of 10, 20, and 30%.

In addition two sets of runs were obtained using all 6 approach volumes, a constant opposing volume of 625 vph and all 3 percent left turns while varying:

1. Signal cycle length of 50, 70, and 90 seconds.
2. Percent green time of 40, 50, and 60 percent.

To use this type of chart in design, the designer would first select the proper figure for the opposing volume and traffic signal conditions involved. Then he would use the chart which includes the percentage of left turns in the inside lane of the approach being studied. He would enter the chart with the value of the hourly traffic volume on the approach and read off the probability of each level of delay occurring. For example, assuming an opposing volume of 500 vph, a 70-second cycle length, 50 percent red time, 33 percent left turns and
an approach volume of 500 vph, one would enter Figure 12, Chart III and read the following values:

- Probability of at least one vehicle being delayed = 1.00
- Probability of more than 5 percent being delayed = 0.40
- Probability of more than 10 percent being delayed = 0.10

If his design criterion is that no more than 10 percent of the vehicles in the inside lane should be delayed more than one cycle length, he may decide that since 90 percent of the time this will not occur, left-turn channelization is not warranted. However, if his criterion requires no more than 5 percent to be delayed, he would surely, in this example, require a separate left-turn lane since this amount of delay will be exceeded 40 percent of the time.

Use of the charts in this form is not very convenient unless all conditions for a particular intersection are covered on one chart. It was therefore decided to develop a nomograph of the stepped variety to represent all variables studied on one chart. Only the 5% level-of-delay is presented in this way (Figure 13), since the other two levels did not provide adequate curves for the full range of all parameters. Use of probit analysis (24) permitted the fitting of straight lines to the probability data.

To use the chart in Figure 13 the designer enters the chart with the approach volume in the lower left and intersects the curve for the opposing volume on a horizontal line. He then draws a vertical line to the appropriate left-turn line in the upper left portion of the chart and a horizontal line is followed to the proper signal cycle length line in the lower right part of the chart. This intersection of lines is then connected by a vertical line to the 0.50 G/C value
EXAMPLE OF CHART USE

GIVEN:
- APPROACH VOLUME = 450 VPH
- OPPOSING VOLUME = 500 VPH
- % LEFT TURNS = 10%
- CYCLE LENGTH = 70 SEC.
- G/C = 0.4

ANS: PROBABILITY (5%) = 0.94

FIGURE 13. NOMOGRAPH FOR DETERMINING PROBABILITY OF 5% OF INSIDE LANE VEHICLES BEING DELAYED FOR MORE THAN ONE SIGNAL CYCLE LENGTH.
in the upper right part of the chart. Lastly the probability of delay to greater than 5% of all vehicles in the inside lane is read from the top scale after adjusting for G/C ratio (by horizontal offset), if it is different than 0.50.

The designer must decide what level of probability is critical for the situation he is evaluating. If the combination of approach and opposing traffic volumes, percent left turns, and signal conditions produces a probability in excess of this level, he should provide a separate left-turning lane; otherwise the intersection can be permitted to operate as it is.

Separate Left-Turning Phase Warrant.

Due to the considerable amount of time required to debug the simulation program, there was insufficient time left to develop more than one set of probability curves for a signalized intersection operating with separate left-turn lanes, but without a separate turning phase on the signal. Results of these simulations showed relatively few vehicles in the inside lane delayed more than one signal cycle length. Only for the highest approach volume and 25-35 percent left turns was there a chance for more than 5 percent of the inside-lane vehicles being delayed. The set of curves shown in Figure 14 was developed in the same way as was done for the unchannelized intersection, except that the curves shown are for the percent of vehicles in the separate left-turn lane that are delayed more than one signal cycle length.

A designer would use these charts in much the same way as suggested for the unchannelized intersection (Figure 12). However, he would use
FIGURE 14. PROBABILITY OF DELAYS GREATER THAN ONE CYCLE LENGTH WHEN OPPOSING VOLUME = 625 V.P.H. (±50) [CYCLE LENGTH = 70 SEC.; 50% GREEN TIME]

CHANNELIZED APPROACH

CHART I (5-15%LT)*

CHART II (15-25%LT)

CHART III (25-35%LT)

Probability of Delay

*% Left Turns of Inside Lane Volume Only

Total Approach Volume - VPH (± 50)
a criterion of design that considers some critical delay level for vehicles in the left-turn lane rather than the inside through lane. It is suggested that the total number of vehicles represented by the percent delayed in the separate turning lane be equal to the number represented by the 5 or 10 percent delay level for the inside lane. Such a conversion is shown in Table 6 and these factors are good for all levels of volume. It should be noted that a given number of left-turn vehicles will produce a higher percentage of left turns on an unchannelized approach than on the channelized approach since it is known that a higher percentage of through vehicles will use the inside lane in the latter case. Drivers realize that left turns will not delay their passage through the intersection when left turns are separated.

### TABLE 6

<table>
<thead>
<tr>
<th>Percent of Inside-Lane Volume That is Left-Turning Vehicles</th>
<th>Percent Vehicles Delayed More Than One Signal Cycle</th>
<th>Unchannelized-% of Inside-Lane Volume</th>
<th>Channelized-% of Left-Turn Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-15</td>
<td></td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>15-25</td>
<td></td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>25-35</td>
<td></td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

Length of Separated Left-Turn Lane. A.A.S.H.O. (2) has a procedure for determining the length of left-turn lane required for storage. The
number of left turns per hour are divided by 30 and multiplied by 25 to obtain the distance in feet. Although this procedure will provide sufficient length for the majority of the signal cycles, the length will likely be too short for a number of other cycles.

During the simulation runs that produced the probability curves in Figure 14, the maximum queue length obtained in the left-turn lane during each signal cycle was determined and a frequency distribution was obtained for each run. These frequencies were combined for all 20 runs in each approach volume set to determine the probability curves shown in Figure 15 (for each volume level there were 1,000 cycles to determine these probabilities). Any value taken from these curves is the probability that the queue length will be equal to or greater than the stated value.

To use these charts, the designer must assume some criterion for the percent of the cycles that he will permit the storage capacity of the left-turn lane to be exceeded; a value of 5 or 10 percent would again be a likely choice. The designer would use the chart applicable to the given percent of left-turn vehicles and enter the values of the approach volume and his design criterion. For example if the percent of left turns = 20 and the approach volume is 600 vph, a design criterion of 10 percent would give an expected queue length of 4 or more left-turn vehicles 10 percent of the time. The designer would therefore provide 3 x 25' = 75' of storage space in the left-turn lane. Additional length may be needed for deceleration purposes if the lane must also serve as a speed-change lane.
Model Validation.

Field studies on two unchannelized intersection approaches were conducted to measure vehicle delays for comparison with delays expected from simulation results for similar conditions. In 12 of 14 cases model correctly predicted the level of delay (greater or less than 5% of inside-lane vehicles delayed) observed in the field studies. This 86 percent agreement of the simulation to the real situation seems to constitute suitable proof of the simulation model's validity.
FIGURE 15. QUEUE LENGTH PROBABILITIES FOR SEPARATED LEFT-TURN LANE WHEN OPPOSING VOLUME = 625 V.P.H. (±50)
[CYCLE LENGTH = 70 SEC.; 50% GREEN TIME]

CHART I (5-15%LT)*

CHART II (15-25%LT)

CHART III (25-35%LT)

% Left Turns of Inside Lane Volume Only
CONCLUSIONS AND RECOMMENDATIONS

Conclusions.

Bearing in mind the limited scope of the research herein reported, the following conclusions seem to be in order:

1. This research was successful in defining the characteristics of vehicles turning left across two opposing lanes of traffic at isolated signalized (fixed time) intersections. The specific characteristics have previously been summarized on pages 27 and 28.

2. A simulation model which represents the operation of traffic on a 4-lane, two-way street (with and without separate left-turning lanes) at a fixed-time signalized intersection was successfully formulated, programmed and operated on the IBM 7040 digital computer. This model more nearly represents the real operation of left-turning vehicles in the intersection area than any other model previously reported in the literature.

3. Digital computer simulation was proven to be the only practical way to accumulate sufficient data to define the suggested warrants. Ninety (90) hours of computer time were required to obtain the data needed to develop Figures 12 to 15. Field studies would have required 1,800 hours of observation plus countless hours of tedious data reduction to arrive at comparable end results.

4. Probability curves have been developed to assist the traffic engineer in deciding when to separate left turns from a two-lane
approach to a fixed-time signalized intersection, when to provide a separate phase for the separated left-turn on such an approach, and how long a storage space should be provided for separated left-turn movements.

5. A design criterion that permits no more than 5 percent or 10 percent of all inside-lane vehicles to be delayed longer than one signal cycle length was the most practical criterion for suggesting warrants for separating left-turn movements.

Recommendations.

1. For the intersection-type studied in this research, it is recommended that additional simulation runs be conducted to extend the applicability of the proposed warrants. Such investigations should include further extensions of traffic volumes and signal timing to cover the full range of commonly encountered values. More nomographs should be developed to make utilization of this design method simpler for practicing traffic engineers.

2. Additional field studies should be conducted to further verify the simulation model and the nature of left-turn gap acceptance distributions as well as other characteristics affecting the left-turn subroutine. Furthermore, there is a need to provide an evaluation of the manner in which lane changes are made on a two-lane approach to a signalized intersection.
3. Further field studies and simulation work are needed to define the characteristics and effects of left turns at signalized intersections on two-lane, two-way streets. It would also be desirable to extend these studies to tee-type intersections.


22. Kell, James H., Data obtained by ITTE, University of California, forwarded with letter of August 19, 1961.


