GAP ACCEPTANCE IN THE FREEWAY MERGING PROCESS

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GAP ACCEPTANCE IN THE FREEWAY Merging PROCESS

ABSTRACT

This study is the first phase of a four-year program on freeway merging undertaken by the Bureau of Public Roads to (1) furnish more detailed information on the effect that geometric variables have on the merging of ramp traffic, (2) develop usable distributions of traffic variables for simulation programs, and (3) develop an optimum ramp metering and merging control system. The emphasis in this report is the collection and collation of gap acceptance characteristics.

The theoretical development of models and useful parameters for describing the merging process include (1) the derivation of the forms of the mean and variance of the delay to a ramp vehicle in position to merge and (2) the treatment of the variability of critical gaps and gap acceptance among drivers through the identification of the representative forms for both critical gap distributions and gap acceptance functions.

Through the application of "individual record probit analyses;" simple, statistically significant relations between the percent gap acceptance and gap size is established. Using this approach, the characteristics of lags and gaps and single and multiple entry merges are compared, as well as fast to slow moving merging vehicles. The probit analyses are generalized to establish a relationship between percent acceptance as the dependent variable and gap size and vehicular speed as the dependent variables.

The fact that 32 ramps—chosen to reflect diverse operating, geometric, geographic and environmental conditions—were continuously filmed at 5 frames per second for an average of an hour, and that enough data was collected to run 1344 usable gap acceptance regressions serve to demonstrate not only the vast quantity of data involved, but the nature of the characteristics now available to interested researchers.
Contents of the Final Report on "Gap Acceptance and Traffic Interaction in the Freeway Merging Process"


Report 430-1: "A Nationwide Study of Freeway Merging Operations"

Report 430-2: "Gap Acceptance in the Freeway Merging Process"

Report 430-3: "Operational Effects of Some Entrance Ramp Geometrics on Freeway Merging"

Report 430-4: "The Determination of Merging Capacity and Its Application to Freeway Design and Control"

Report 430-5: "Traffic Interaction in the Freeway Merging Process"

Report 430-6: "Digital Simulation of Freeway Merging Operation"

Report 430-7: "Annotated Bibliography on Gap Acceptance and Its Applications"

The opinions, findings and conclusions expressed in these publications are those of the authors and not necessarily those of the Bureau of Public Roads.
INTRODUCTION

Scope of the Project

The subject of ramp vehicles merging into the freeway stream has deservedly been treated by a number of researchers. Most of the research has been devoted to empirical studies leading to design and operational procedures. Mathematical treatment of the merging maneuver has been attempted, too, with somewhat limited success because of the complexity of the vehicle interactions. Computer technologists have contributed several digital computer simulation programs, but lack of detailed criteria on gap acceptance and merging logic has hampered progress.

In the summer of 1965, the U.S. Bureau of Public Roads undertook research to furnish detailed criteria on the merging of ramp vehicles into the freeway system. A contract, "Gap Acceptance and Traffic Interaction in the Freeway Merging Process," was awarded to the Texas Transportation Institute. The general aim of this research is the conception of a relationship between the many variables associated with the interaction of vehicles traversing a ramp and merging onto a freeway so as to determine the effects of the following on merging operation and level of service:

(1) Traffic characteristics such as gap availability, gap acceptance, speed and volume;

(2) Ramp geometrics such as length, curvature, angle of convergence and grades, and acceleration lane geometrics such as length, shape, delineation and location of lateral obstructions;

(3) System considerations such as interchange type, ramp configurations, frontage roads and upstream or downstream bottlenecks, and environmental elements such as metropolitan area size, location within the city, and lighting;

(4) Control devices such as freeway lane controls, yield or merge-ahead signs, traffic signal feeding the entrance ramp, and ramp metering stations.

The underlying purpose of this research is the application of the above information to the following:
(1) In design and operation--the furnishing of more-detailed information on the effect that geometric variables and traffic characteristics have on merging traffic;

(2) In simulation--the development of usable distributions of traffic variables for simulation programs.

In order to fulfill the broad project objectives, 32 ramp-freeway connections located in 8 metropolitan areas in 6 states from coast to coast and from border to border were chosen. These locations--specially chosen to give a complete range of geographic, environmental, geometric, and operating conditions--are summarized in Appendix A.

Specific Objectives

There are three purposes for conducting field studies of traffic characteristics in ramp-freeway merging areas. The first reason is for the eventual testing and refinement of models. The second purpose for collecting data is that at the present time only very limited data of this type are available. Gap acceptance data is a prime example of this. Much of the meager gap acceptance data available is very old data, based on a small sample size, or for a peculiar situation such as a left-hand ramp or stop sign control. These conditions severely limit the usefulness of these data. No data has been collected on the effect of ramp and acceleration lane geometrics on the gap acceptance characteristics of vehicles on the ramp. A third very important application of the data on merging characteristics is for simulation inputs.

Specific objectives of this phase of the project research are:

(1) Development of models and useful parameters for describing gap and lag acceptance and delay in the merging maneuver;

(2) Presentation of gap and lag acceptance characteristics obtained from studying some 32 ramps across the United States.

(3) Determination of any differences in gap acceptance characteristics for different points of entry along the acceleration lane.

(4) Delineation of the roles of absolute and relative speeds in the merging process and their effect on gap acceptance.
(5) Identification of gap acceptance characteristics of more than single ramp vehicles so as to determine the efficiency of platoon merging;

(6) Investigation of the effect of outside freeway lane volumes on gap acceptance.
THEORY

Definitions and Terminology

In actual practice, the entrance ramp-freeway connection may be regarded as a special case of the uncontrolled intersection. The three fundamental maneuvers performed by vehicles in the vicinity of this connection area may be identified as follows:

Lane Change. The transfer of a vehicle from one traffic lane to the next adjacent traffic lane.

Merging. The process by which vehicles in two separate streams moving in the same general direction combine or unite to form a single stream.

Weaving. The oblique crossing of one stream of traffic by another accomplished by the merging of the two streams into one and then the diverging of this common stream into separate streams again.

The freeway elements associated with the above maneuvers are:

Acceleration lane. An added width of pavement adjacent to the main roadway traffic lanes enabling vehicles entering the main roadway to adjust their speed to the speed of through traffic before merging.

Ramp. A connecting roadway between two intersecting or parallel roadways, one end of which joins in such a way as to produce a merging maneuver.

Frontage road. A roadway paralleling a freeway so as to intercept traffic entering or leaving the facility and to furnish access to abutting property.

Basic to the description of traffic interaction in the merging process are the following variables:

Headway. The interval of time between successive vehicles moving in the same lane measured from head to head as they pass a point on the road.

Space headway. The distance between successive vehicles moving in the same lane measured from head to head at a given instant in time.
Gap. A major stream headway that is evaluated by a minor stream vehicle desiring to either merge into or cross the major stream. The units may be those of either time or distance (space gap).

Lag. The interval of time between the arrival of a minor stream vehicle and the arrival of a major stream vehicle at a reference point or points in the vicinity of the area where the streams either cross or merge.

Space lag. The distance between a minor stream vehicle and a reference point where the minor stream crosses or joins a major stream, subtracted from the distance between a major stream vehicle and the same reference point, with both distances measured at a given instant in time.

Figure 1 is a time-space diagram prepared to illustrate the relationship between the geometric elements comprising the merging area and the movements of mainstream and ramp vehicles within the area. Distance is plotted as the ordinate, time is the abscissa, and the slope of the traces denote speed. The traces for freeway and ramp vehicles are identified in the figure. A vehicle is regarded as a ramp vehicle as long as it remains at least partially on the acceleration lane.

It is convenient to subdivide merges into their basic types and classifications, then to determine the kind and amount of information needed for intelligent analysis of the movements. Following is a listing of merge maneuvers, defined for use in this report: (see Figure 2)

Optional merge. The merging vehicle voluntarily moves from the acceleration lane into the outside traffic lane.

Confined merge. The merging vehicle is forced into the outside traffic lane by the presence of the end of the acceleration lane.

Ideal merge. The merging vehicle is able to enter the freeway stream without causing a freeway vehicle to reduce its speed or change lanes.

Forced merge. The ramp vehicle effects the merging maneuver into the freeway stream so that an oncoming freeway vehicle or vehicles must either slow down or change lanes.

Single entry merge. One ramp vehicle moves into a single freeway time gap.
TIME-SPACE RELATIONSHIPS OF FREEWAY MERGING MANEUVER

FIGURE 1
TYPES OF FREEWAY MERGES

Figure 2
Multiple entry merge. Two or more ramp vehicles merge into a single freeway time gap.

From Figures 1 and 2 and the above classification, it is apparent that a merge must be qualified as being either single or multiple entry, optional or confined, ideal or forced, and gap or lag. For example, a given merge might be described as being a "single entry, optional, ideal gap merge."

**Merging Parameters**

In the previous section, a few traffic variables—headways, gaps and lags—were identified. In addition to these variables, the speed of the major stream vehicles, speed of the merging vehicles, relative speed, major stream flow, and minor stream flow are additional variables that must be considered in any rational description of the merging process.

Whereas a variable is a quantity which can assume any value or number, a parameter is a term used in identifying a particular variable or constant other than the coordinate variables. For example, "volume" is a traffic variable and "capacity"—the maximum volume that a facility can accommodate—is the corresponding traffic parameter. Some important figures of merit for describing the gap acceptance phenomenon in freeway merging are the critical gap, percent of ramp vehicles delayed, mean duration of static delay accepting a gap, mean length of queue, and total waiting time on the ramp.

Several "critical" values have been discussed in the literature. Greenshields\(^1\) defined the "acceptable average-minimum time gap" as a gap accepted by half the drivers. Raff\(^2\) used a slightly different parameter, the "critical lag." The critical lag is the size lag which has the property that the number of accepted lags shorter than the critical lag is equal to the number of rejected lags longer than the critical lag. As such the Greenshields and Raff parameters are median values.

The principal use of gap acceptance parameters is to simplify the computation of the delay duration by permitting the assumption that all intervals shorter than the critical value (lag or gap) are rejected while all intervals longer are accepted. It has been suggested\(^3,4\) that the mean of the critical gap distribution, not the median should be used in delay computations. Nevertheless, the "median critical gap" remains a practical parameter because, as shall be shown, it can be readily obtained graphically.
Delay Models

There have been a number of theoretical papers dealing with the delay to a single waiting vehicle on the minor street of an uncontrolled intersection, due to the traffic on the outside lane of an intersecting highway. It can be shown that this theory is valid for a driver on a ramp waiting to join a stream of traffic.

Most discussions of this problem have assumed that the distribution of main stream arrivals is Poisson, i.e., that the probability that a given gap is between \( t \) and \( t + dt \) seconds is given by an expression of the form \( q e^{-qt} \) where \( q \) is the flow. Raff considered the delay problem as related to vehicles whereas Tanner's analysis was specifically applied to pedestrian delays. The pedestrians were assumed to arrive at random, and all waited for a critical time gap of \( T \) seconds in order to cross the highway. Although Mayne showed that one could obtain results for the Laplace transform of the delay duration for other than Poisson traffic on the main stream traffic, the only case considered in detail was for this headway distribution. The derivation which follows assumes Erlang headways and as such is a generalization of the combinatorial techniques suggested by Raff, Tanner and Mayne.

It may be assumed that a ramp driver waiting to merge measures each time gap, \( t \), in the traffic on the outside lane of the freeway until he finds an acceptable gap, \( T \), which he believes to be of sufficient length to permit his safe entry. If he accepts the first gap (\( t > T \)), his waiting time is zero. If he rejects the first gap (\( t < T \)), but accepts the second gap, his expected waiting time would be one gap interval. If it is assumed that the driver's gap acceptance does not change with time, then by induction, the individual waiting periods form a geometric distribution and the probability, \( P(n) \), of any driver having a wait for \( n \) intervals each less than \( T \) seconds before merging is

\[
P(n) = p^n (1-p), \quad n = 1, 2, \ldots \tag{1}
\]

where

\[
p = P(t < T) = \int_0^T f(t) \, dt \tag{2}
\]

and \( f(t) \) is the distribution of gaps in the major stream. It is convenient to define a normalized random variable \( w \) such that

\[
w = t / T \tag{3}
\]
or
\[ t = T w \]  \hspace{1cm} (4)

and
\[ dt = T \, dw \]  \hspace{1cm} (5)

Substituting (4) and (5) into (2) yields
\[ p = P(w < 1) = \int_0^1 f(T w) \, T \, dw. \]  \hspace{1cm} (6)

The density of w may be defined as
\[ g(w) = f(T w) \, T, \]  \hspace{1cm} (7)

which when inserted in (6) gives
\[ p = \int_0^1 g(w) \, dw. \]  \hspace{1cm} (8)

If the distribution of gaps on an outside freeway lane of flow q can be described by the Erlang distribution,
\[ f(t) = \frac{(aq)^a}{\gamma(a)} t^{a-1} e^{-aq t} ; \]  \hspace{1cm} (9)

then it follows from (3) to (8) that
\[ p = \int_0^1 \frac{(aqT)^a}{\gamma(a)} w^{a-1} e^{-aqTw} \, dw. \]  \hspace{1cm} (10)

Equation (10) is of the form of the incomplete gamma distribution which is of course equivalent to the cumulative Poisson:
\[ p = 1 - e^{-aqT} \sum_{i=0}^{a-1} (aqT)^i/i! \]  \hspace{1cm} (11)

and
\[ 1 - p = e^{-aqT} \sum_{i=0}^{a-1} (aqT)^i/i! \]  \hspace{1cm} (12)

Placing (11) and (12) in (1) gives the probability that the first n time-gaps between vehicles in the major stream are all less than the normalized critical gap, \( w = 1 \) but that the \( n + 1 \)th is greater than the normalized critical gap. This probability \( P(n) \) averaged over the distribution of gaps less than the normalized critical gap, \( g(w) \) \( (0 < w < 1) \), gives the distribution of waiting time or delay \( f(µ) \) to a ramp.
vehicle in position to merge. In terms of the moment generating functions this may be expressed as

$$M_\mu(\theta) = \sum_{n=0}^{\infty} P(n) M_w(\theta)^n,$$  \hspace{1cm} (13)

where $M_\mu(\theta)$ is the m. g. f. of the distribution of delay to a ramp vehicle waiting for a gap greater than the normalized critical gap. From (1), (11) and (12), it is apparent that

$$M_\mu(\theta) = (1 - p) \sum_{n=0}^{\infty} \left[ p M_w(\theta) \right]^n \hspace{1cm} (14)$$

$$= \frac{1 - p}{1 - p M_w(\theta)} \hspace{1cm} (15)$$

In order to use (15), $M_w(\theta)$ must be found. It is apparent that $g(w)$ ($0 < w < 1$) is a conditional probability which by definition becomes

$$g(w) (0 < w < 1) = P(w < w | w < 1) \hspace{1cm},$$

$$\int_{0}^{w} g(w) \, dw = \frac{\int_{0}^{1} g(w) \, dw}{\int_{0}^{1} g(w) \, dw} \hspace{1cm} (16)$$

Since the denominator of (16) is given is (8), one obtains the density of gaps less than the normalized critical gap as

$$g(w) (0 < w < 1) = g(w)/p \hspace{1cm},$$

and the m. g. f. as

$$M_w(\theta) = \frac{1}{p} \int_{0}^{1} e^{\theta w} g(w) \, dw \hspace{1cm} (17)$$

Changing the limits of summation and substituting for $g(w)$ yields

$$M_w(\theta) = \frac{(aqT)^a}{p^\gamma(a)} \left[ \int_{0}^{\infty} w^{-a-1} e^{-w(aqT-\theta)} \, dw - \int_{1}^{\infty} w^{-a-1} e^{-w(aqT-\theta)} \, dw \right]$$

If a change of variable is made such that $u = w(aqT-\theta)$ in the first integral and $u + 1 = w$ in the second integral, then

$$M_w(\theta) = \frac{(aqT)^a}{p^\gamma(a)} \left[ (aqT-\theta)^{-a} \int_{0}^{\infty} u^{-a-1} e^{-u} e^{-(aqT-\theta)} \, du - (u + 1)^{-a} \int_{0}^{\infty} e^{-u(aqT-\theta)} \, du \right]$$

11
Use of Newton's binomial identity,
\[(u + 1)^{a-1} = \sum_{i=0}^{a-1} \binom{a-1}{i} u^{a-i-1},\]
in the second integral, and noting that the first integral is a gamma function gives
\[
M_\mu(\theta) = \frac{1}{p} \left( 1 - \frac{\theta}{aqT} \right)^{-a} - \frac{(aqT)^a}{p\gamma(a)} e^{-(aqT-\theta)} \sum_{i=0}^{a-1} \binom{a-1}{i} \int_0^\infty u^{a-i-1} e^{-u(aqT-\theta)} du. \tag{19}
\]
Since the second integral is a gamma function with parameters \((a-i)\) and \((aqT-\theta)\), the second term may be written as
\[
\frac{(aqT)^a}{p\gamma(a)} e^{-(aqT-\theta)} \sum_{i=0}^{a-1} \frac{\gamma(a)}{i!\gamma(a-i)} \frac{\gamma(a-i)}{(aqT)^{a-i}}
\]
or
\[
\frac{aqT}{aqT-\theta} e^{-(aqT-\theta)} \sum_{i=0}^{a-1} \frac{(aqT-\theta)^i}{i!} \tag{20}
\]
Substituting (20) for the second term in (19) and collecting terms, one obtains
\[
M_\mu(\theta) = \frac{1}{p} \left( 1 - \frac{\theta}{aqT} \right)^{-a} \left[ 1 - e^{-(aqT-\theta)} \sum_{i=0}^{a-1} \frac{(aqT-\theta)^i}{i!} \right]. \tag{21}
\]
Now it is possible to find the moments of the delay to a ramp vehicle in position to merge by evaluating the derivatives of (15),
\[
\frac{dM_\mu(\theta)}{d\theta} = \frac{p (1-p)}{[1-pM_\mu(\theta)]^2} \frac{dM_\mu(\theta)}{d\theta} \tag{22}
\]
and
\[
\frac{d^2M_\mu(\theta)}{d\theta^2} = \frac{p(1-p) [dM_\mu(\theta)/d\theta]^2}{[1-p M_\mu(\theta)]^2} + \frac{2(1-p) [pdM_\mu(\theta)/d\theta]^2}{[1-p M_\mu(\theta)]^3}, \tag{23}
\]
at \(\theta = 0\), where the derivatives of \(M_\mu(\theta)\) in the expressions are given by
\[
\frac{dM^n_w(\theta)}{d\theta} = \gamma(a+n) \frac{(aqT)^a e^{-aqT-\theta}}{p_\gamma(a)} \left[ e^{(aqT-\theta)} - \sum_{i=0}^{a+n-1} \frac{(aqT-\theta)^i}{i!} \right].
\] (24)

Expansion of (22) and (23) leads to the following expressions for the mean and variance of the delay distribution:

\[
\mu(w)_a = \frac{e^{aqT} - \sum_{i=0}^{a} \frac{(aqT)^i}{i!}}{qT \frac{a+1}{\ell} \frac{(aqT)^i}{i!}}
\] (25)

\[
\sigma^2(w)_a = \frac{(a+1) [e^{aqT} - \sum_{i=0}^{a+1} \frac{(aqT)^i}{i!}]}{a(qT)^2 \frac{a-1}{\ell} \frac{(aqT)^i}{i!}} + \mu^2(w)_a
\] (26)

Converting from the normalized parameter \( w \) to the original variable \( t \), the mean and variable for values \( a = 1, 2, 3 \) and \( 4 \) become:

\[
\mu(t)_1 = q^{-1} (e^{qT} - 1 - qT)
\] (27)

\[
\mu(t)_2 = \frac{e^{2qT} - 1 - 2qT - 2(qT)^2}{q(1+2qT)}
\] (28)

\[
\mu(t)_3 = \frac{e^{3qT} - 1 - 3qT - 4.5(qT)^2 - 4.5(qT)^3}{q [1 + 3qT = 4.5(qT)^2]}
\] (29)

\[
\mu(t)_4 = \frac{e^{4qT} - 1 - 4qT - 8(qT)^2 - 10.7(qT)^3 - 10.7(qT)^4}{q [1 + 4qT + 8(qT)^2 + 10.67(qT)^3]}
\] (30)

\[
\sigma^2(t)_1 = \frac{e^{2qT} - 2qT e^{qT} - 1}{q^2}
\] (31)

\[
\sigma^2(t)_2 = \frac{2e^{4qT} - e^{2qT} - 2qT e^{2qT} - 8(qT)^2 e^{2qT} - 1 - 4qT - 2(qT)^2}{2q^2 (1+2qT)^2}
\] (32)
\[ \sigma^2(t)_3 = \frac{4e^{3qT} - 12T - 18(qT)^2 - 18(qT)^3 - 13.5(qT)^4}{3q^2 [1 + 3qT + 4.5(qT)^2]} + \mu^2(t)_3 \]  
\[ \sigma^2(t)_4 = \frac{5e^{4qT} - 50T - 40(qT)^2 - 53.3(qT)^3 - 53.3(qT)^4 - 42.67(qT)^5}{4q^2 [1 + 4qT + 8(qT)^2 + 10.67(qT)^3]} + \mu^2(t)_4 \]  

Equations 27-30 are plotted in Figure 3 and Equations 31-34 in Figure 4.

**Critical Gap Distributions**

The theoretical delay values of the previous section are based on a fixed critical gap for all drivers. A more realistic description of delays can be obtained by replacing the fixed critical gap with a distribution of critical gaps, \( f(T) \). The forms for the mean and variance of the distribution of delay become

\[ M(T) = \int_0^\infty \mu(T) f(T) \,dT \]  
\[ S^2(T) = \int_0^\infty \sigma^2(T) f(T) \,dT \]  

Assuming that the headway distribution on the freeway is negative exponential, for example, substitution of (27) in (35) yields

\[ M(T) = q^{-1} \int_0^\infty e^{qT} f(T) \,dT - \int_0^\infty T f(T) \,dT - q^{-1} \int_0^\infty f(T) \,dT. \]  

Realizing that the second term defines the mean of the critical gap distribution and the integral in the last term must equal unity gives

\[ M(T) = q^{-1} \int_0^\infty e^{qT} f(T) \,dT - \tilde{T} - q^{-1} \]  

Some representative forms for critical gap distributions are shown in Figure 5. If one assumes that the critical gaps of the drivers are distributed uniformly between \( c \) and \( c_1 \) then
Merging delay in terms of the freeway flow $q$, critical gap $T$, and Erlang constant, $\alpha$.  

Figure 3
VARIANCE IN DELAY IN TERMS OF THE FREEWAY FLOW $q$, CRITICAL GAP $T$, AND ERLANG CONSTANT $\alpha$.

FIGURE 4
RECTANGULAR DISTRIBUTION

\[ f(t) = \frac{1}{c_1 - c} \]

TRANSLATED NEGATIVE EXPONENTIAL

\[ f(t) = \frac{1}{t-c} e^{-(t-c)/(\bar{T}-c)} \]

ERLANG DISTRIBUTION

\[ f(t) = \frac{(a/\bar{T})^a}{(a-1)!} t^{a-1} e^{-at/\bar{T}} \]

LOG-NORMAL DISTRIBUTION

\[ f(t) = \frac{1}{s \sqrt{2\pi}} e^{-\left(\ln t - \ln \bar{t}\right)^2/2s^2} \]

FREQUENCY OF CRITICAL GAPS OF SIZE \( t \)

GAP SIZE, \( t \)

REPRESENTATIVE FORMS FOR CRITICAL GAP DISTRIBUTION:

Figure 5
The translated negative exponential distribution,
\[ f(T) = (\frac{T}{\bar{T}} - c)^{-1} e^{-(T-c)/(\bar{T}-c)} \] 
has also been suggested as a gap distribution function, as has the Erlang distribution, \[ f(t) = \frac{(a/\bar{T})^a}{(a-1)} T^{a-1} e^{-aT/\bar{T}} \]

Substitution of (39), (40) and (41) in (38) gives respectively:
\[ M(T) = (c_1-c)^{-1} q^{-2} \left[ e^{qc} - e^{qc} \right] \frac{1}{\bar{T} - q^{-1}} \] 
\[ M(T) = \left[ q(1-q\bar{T}+qc) \right]^{-1} e^{qc} \frac{1}{\bar{T} - q^{-1}} \] 
and
\[ M(T) = \frac{1}{q} \left( \frac{a}{a-q\bar{T}} \right)^a \frac{1}{\bar{T} - q^{-1}} \]
which correspond to the distribution of delay for the first three critical gap distributions in Figure 5.

The derivations of this section (Equations 35-44) are based on utilization of a distribution of critical gaps for all drivers, \( f(T) \). This is a concession to the obvious fact that not all drivers have the same critical gap. The difficulty lies, however, in measuring the critical gap for individual drivers in order to obtain a distribution of critical gaps. One technique to obtain such a frequency distribution samples only drivers who have rejected at least one gap before merging and is based on the reasonable assumption that the driver's critical gap must lie somewhere between the largest gap he rejected and the gap he finally accepted.

Although observations of a narrow range delimit a driver's gap more closely, by the very nature of the technique observations with wider ranges have greater influence on the shape of the density function. Dawson suggests two weighting procedures to overcome this shortcoming which seems promising. Still, there exist some practical as well as conceptual shortcomings in the technique which tended to rule it out as a basis of parameter determination for this project.
First, the technique only considers drivers who have rejected a gap; and secondly, the technique is obviously not applicable to lags.

**Gap Acceptance Functions**

In the derivations in the preceding sections based on the critical gap concept, it is assumed that the waiting driver evaluates each inter-car time gap, choosing to merge if the gap is greater than some predetermined time gap T, or not, if the gap is less than T seconds. Herman and Weiss\textsuperscript{13} suggest that a more realistic model would be to associate with each time gap, a gap acceptance probability $P(T)$ such that the waiting driver crosses the highway with the probability $P(T)$ when confronted with a gap of duration T. Some representative gap acceptance functions are illustrated in Figure 6.

Using methods strongly dependent on renewal theory, Weiss, and Maradudin\textsuperscript{14} formulate the merging delay problem in terms of an integral equation instead of using the combinatorial reasoning approach inherent in Equations 1 through 44. The expression for the mean of the delay distribution is:

$$M(T) = \int_{0}^{\infty} T \psi_o(T) dT + \frac{1-P_o}{P} \int_{0}^{\infty} T \psi(T) dT$$  \hspace{1cm} (45)$$

where

$$P_o = \int_{0}^{\infty} P(T) f_o(T) dT$$  \hspace{1cm} (46)$$
$$P = \int_{0}^{\infty} P(T) f(T) dT$$  \hspace{1cm} (47)$$

$$\psi_o(T) = f_o(T) [1-P(T)]$$  \hspace{1cm} (48)$$
$$\psi(T) = f(T) [1-P(T)]$$  \hspace{1cm} (49)$$

and $f(T)$ is the distribution of gaps on the outside freeway lane and $P(T)$ is the gap acceptance function. The zero subscripts refer to the gap availability distribution and gap acceptance function for the first gap.

Weiss\textsuperscript{13,14} has applied his theory to some specific distributions.
TRAPEZOIDAL FUNCTION

\[ P(t) = \begin{cases} 
0 & (t < c) \\
(t-c)/(c_1-c) & (c < t < c_1) \\
1 & (t > c_1) 
\end{cases} \]

TRANSLATED NEGATIVE EXPONENTIAL

\[ P(t) = \begin{cases} 
1 - e^{-(t-c)/(\bar{t}-c)} & (t > c) \\
0 & (t < c) 
\end{cases} \]

ERLANG FUNCTION \((a=1,2,\ldots)\)
NEG. EXPONENTIAL \((a=1)\)
STEP FUNCTION \((a=\infty)\)

\[ P(t) = \begin{cases} 
0 & (t < \bar{t}) \\
1 & (t > \bar{t}) 
\end{cases} \]

\[ P(t) = \frac{(a/\bar{t})^a}{(a-1)!} \int_0^t a^{a-1} e^{-at/\bar{t}} \, dt \]

LOG-NORMAL FUNCTION

\[ P(t) = \frac{1}{s\sqrt{2\pi}} \int_0^t \frac{e^{-\left(\ln t - \ln \bar{t}\right)^2/2s^2}}{t} \, dt \]

REPRESENTATIVE FORMS FOR GAP ACCEPTANCE FUNCTIONS

Figure 6

20
For example if the distribution of gaps on the outside lane of the freeway is taken as
\[ f(T) = f_0(T) = q e^{-qT} \]  
and the gap acceptance function is
\[ P(T) = P_0(T) = 1 - e^{-\lambda(T-c)} \]
one obtains for the mean delay to a ramp vehicle in position to merge
\[ \text{M(T)} = q^{-1} \left[ e^{qc} - 1 - qc + \frac{q}{\lambda} \right. \]
\[ + \left. \left[ \frac{q}{\lambda(q+c)} + qc \right] e^{-qc} \right] \]
(52)

It is interesting to note that Equation 27 is, as one would expect, a special case of both (44) and (52). Taking (44) first,
\[ \lim_{a \to \infty} (1 - \frac{q \bar{T}}{a})^{-a} = e^{q \bar{T}} \]
(53)
from the definition of "e", the base of the natural system of logarithms. Since the variance of the Erlang distribution is zero for \( a = \infty \), this may be interpreted as the case for which the critical gap in (41) is constant and has the value \( \bar{T} \). Substitution of (53) in (44) gives (27).

Likewise in (52) since \( \lambda = (\bar{T}-c)^{-1} \) (see Figure 6), if \( \bar{T} = c \) we have a step function for the gap acceptance probability and (52) also reduces to (27).

The fact that the delay obtained using the fixed value is both a special case of the delay obtained using the critical gap distribution in (41) and a special case of the gap acceptance function in (51) tends to establish (1) that the probability of a driver accepting a gap of size \( T \), \( P(T) \) in Figure 6, is the same as the probability of that driver having a critical gap less than \( T \) (see Figure 5),
\[ P(T) = \int_0^T f(t) \, dT \]
(54)
and (2) that the mean not the median of both the critical gap distribution or the gap acceptance function is the correct parameter for delay computations.

Theoretically then, the critical gap may be obtained from a gap acceptance function. From a practical point of view this is desirable because it is much easier to obtain data on the gap acceptance characteristics of drivers than it is to try to measure the critical gaps of drivers directly. Moreover⁴, it has been shown that the mean of the critical gap distribution and the mean of the gap acceptance function for a given ramp do in fact exhibit close agreement.
PROCEDURE

Data Collection

Since data on merging characteristics was to be collected at some 32 ramp locations from the far corners of the United States, there was a definite need for a standardized method of data collection in order to facilitate both the collection and analysis of data. The simultaneous collection of the many variables pertinent to the overall project objectives required the continuous viewing of an area of influence of about a quarter of a mile.

An aerial photographic technique was developed utilizing a 35 mm Automax data recording camera attached to a special tripod which was mounted in the rear of a Cessna P206 aircraft. The rear baggage compartment door was fitted with a large plexiglass window. The aircraft was circled above each entrance ramp in a radius of about 1/4 to 1/2 mile during the peak traffic period and the traffic was filmed at 5 frames per second. Two 400 ft. magazines were used with the camera which allowed two filming periods of 22 minutes each with approximately 2 minutes in between to switch magazines. The camera was fitted with a data chamber which included a clock, a frame counter, and a data slate where information about the study location could be recorded. The data chamber information was automatically recorded on one edge of the film.

Data Reduction

The frame number in which each vehicle in the outside lane of the freeway and in which each ramp vehicle crossed 200 ft. stations (marked on the shoulder by 1 by 6 ft. plastic stripes so as to be seen clearly in the films) was recorded. In this way, gaps could be determined at any point within the 1/4 mile study area to an accuracy of 1/5 second.

A battery of programs was written. These include a time-space diagram of the merging area for the entire study period using the Cal-Comp plotter of the Texas A&M University Data Processing Center; plots of contour maps of such descriptive variables as speed, volume, density, acceleration noise, speed noise, energy, and shock wave speed; and summaries of volume-speed-density relations. These are discussed in detail in another report.
Pursuant to the objectives of this particular report, a computer program was written which identifies for each ramp vehicle, the rejected (or accepted) lag, all rejected gaps, the gap finally accepted, the gap after the one accepted, the delay to the ramp vehicle, and the station of entry in the acceleration lane. A sample of this output is illustrated in Appendix B. All lags and gaps are referenced to the ramp nose.

The Binomial Response

If an entrance ramp driver, selected at random from a population, is given an opportunity to merge, the probability he will accept is \( p \); the probability to rejecting the opportunity is \( (1-p) \) or \( q \). The opportunity to merge here may be measured by the time gap, space gap, relative speed, or some combination of these or other factors. If two drivers are given the same opportunity, and if their reactions are completely independent, the probability that both accept is \( p^2 \), and the probability that both reject is \( q^2 \); the probability that only the first accepts is \( pq \), and the probability that only the second accepts is \( qp \). Thus the total probabilities of 2, 1, and 0 accepting are \( p^2 \), \( 2pq \) and \( q^2 \) respectively, the successive terms in the expansion of \( (p+q)^2 \). In a similar manner it may be seen that if a group of \( n \) drivers is exposed to the same merging conditions, and all react independently, the probabilities of \( n \), \( (n-1) \), \( (n-2) \), \ldots, 2, 1, 0 responding are the \( (n+1) \) terms in the binomial expansion \( (p+q)^n \). The probability of exactly \( x \) acceptances is therefore

\[
P(x) = \binom{n}{x} p^x q^{n-x}
\]

Equation (55) of course represents the Binomial Distribution of probabilities. The average number of gaps accepted in \( n \) opportunities is \( np \) and the average number rejected is \( nq \). The variance of the distribution is \( npq \).

It is apparent that the gap acceptance phenomenon involves a "stimulus" (available time gap) applied to a "subject" (the driver of a ramp vehicle). Variation of the stimulus is followed by a change in some measurable characteristic associated with the subject. This measurable characteristic--the acceptance of a certain gap, in the case at hand--is referred to as the "response" of the subject.

Methods employed for the estimation of the nature of a process by means of the reaction that follows its application to living matter are
called "biological assays." In its widest sense the term should be understood to mean the measurement of any stimulus (physical, chemical, biological, physiological, or psychological) by means of the reactions which it produces. One type of assay which has been found useful in many different disciplines is that dependent on the all or nothing response. The decision to accept or reject a gap is the type of response which permit no graduation and which can only be expressed as occurring or not occurring. The statistical treatment of this particular assay has been greatly facilitated by the development of probit analysis.

The Probit Method

Probit analysis is a well-established technique used widely in toxicology and bioassay work. Reference is made to two books by D. J. Finney in which, in the first, the technique is described and a rather lengthy computational method set forth; in the second many examples covering a wide variety of cases are presented and the underlying statistical theory presented in some detail.

Normally, in an experiment to which a probit analysis is applied, the magnitude of the stimulus is controlled by the experimenter. Thus, a number of subjects (usually insects) are administered a stimulus (usually a poison) to which they either do or do not exhibit a certain response (usually dying). Each subject is assumed to have a tolerance for the stimulus; if the stimulus is greater then the subject responds (insect dies), etc. Typically what is desired is a relationship expressing the percent killed as a function of the dose. Since this is not a linear relationship the estimation of the equation of the curve and tests of significance are severely complicated. The transformation from percentages to "probits" forces the curve into a linear relationship.

In a study of lag and gap acceptances at stop-controlled intersections, Solberg and Oppenlander showed that the probit of the percent accepting a time gap is related to the logarithm of the time gap \( x \) by the equation

\[
Y = a + bx
\]

By means of the probit transformation the study data were used to obtain an estimate of this equation. The parameters of the tolerance distribution, mean and variance, were also determined. In particular the median gap and lag acceptance times were easily estimated from that value of \( x \) when \( Y=5 \) (percent acceptance is 50%).
Whereas, Solberg and Oppenlander tabulated the data into groups at 1-second intervals in order to obtain an estimate of drivers accepting this interval in the time series, in this report the data were not grouped. Such groupings are usually reserved for experiments in which the magnitude of the stimulus is controlled. In traffic studies such control is not possible though the magnitude of the stimulus (available gap size) may be measured and recorded along with the response (accepted or rejected). Finney terms such data "individual records." The probit method is just as applicable to individual records as to data from a controlled (grouped) experiment.

In the present study there were two practical considerations for not grouping the data. First, in the Solberg-Oppenlander data for intersections the effective range of accepted and rejected gaps was from about 2 to 13 seconds, however in the present investigation for many ramps studied the range was as low as from 1 to 3 seconds. Secondly, the wide geographic distribution of the thirty-five study sites and the relative travel made it difficult to film for an adequate duration at each location in order to obtain representative samples. It was quickly ascertained that, for grouping, an interval of .2 sec. would be needed to give enough groups and several hours of data would be needed at each ramp to obtain enough gap data for each .2 sec. interval.

There are, of course, problems associated with the individual records approach used here. As Finney notes, the chi-squared statistics computed in the statistical tests of significance are not as reliable as with the grouped data. Another problem with the individual records is that the iterative solution technique employed may fail to converge in cases where the sample size is small or where the data are particularly irregular.

The development of the probit analysis for individual records is made in Appendix C.
RESULTS OF SINGLE VARIABLE ANALYSIS

Notation and Format

It is important to remember that what is desired is the form of the gap (or lag) acceptance function for the freeway merging maneuver. Many theoretical forms of this function were described in the section on "Theory" and are illustrated in Figure 6. Several researchers have shown that the log-normal function provides the best description of the gap acceptance phenomena. The purpose of the probit analysis is to transform this log-normal function into the linear form

\[ Y = a + bx \] (55)

where \( x = \log t \), \( t \) being the time interval (either lag or gap as the case may be) and \( Y \) is the probit of \( P \), \( P \) being the probability of accepting a gap or lag of \( t \) (see Figure 6d).

If \( x \) and \( Y \) are plotted on a grid, Equation 55 has the form of a straight line. For convenience then, log-probability graph paper has been used throughout this section so that the relationship between \( t \) and \( P \) from Figure 6d may be viewed as a straight line. The abscissa of this graph paper (see Figure 7) is in the form of a logarithmic scale denoting the gap (or lag) interval \( t \) in seconds; the right ordinate is a probability scale denoting Percent Acceptance \( P \) and the left ordinate is a linear scale denoting the Probit of Acceptance \( Y \). Thus the abscissa provides the transformation between \( x \) and \( t \) and the ordinate establishes the relationship between \( Y \) and \( P \).

In the discussion which follows, a "zero" subscript will be used for lags and a "one" will be used in the subscript for gaps.

Gaps and Lags

In the past, some investigators have chosen to work with gaps, some with lags, and some with both. The choice of variable should not, of course, be arbitrary. Ramp drivers approaching the freeway merging area evaluate lags. In many cases, the driver evaluates the lag rather than the first gap since the lead vehicle on the freeway may have passed long before the ramp driver was in position to merge.

The reason then for studying gaps as well as lags is a practical one. It is anticipated that one important application of this research will be in
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 7
the eventual development of a merging control system to help drivers execute this difficult maneuver. Presumably, such a system will be based on fitting merging ramp vehicles into openings in the freeway stream. This is most easily accomplished by a gap detector on the outside lane of the freeway located far enough upstream from the merging area to allow a metered ramp vehicle to reach the merging area at the same time as an acceptable gap.

In Figure 7, the correspondence between the effects of lag size and gap size on acceptance may be seen. Illustrated are curves for 12 films for 7 ramps taken in San Francisco (designated SF in the Figure), Sacramento (SC), and Los Angeles (LA). The two non-parallel dashed lines (designated "unconstrained" in the lower right corner of the Figure) define the regression of acceptance on lags and acceptance on gaps. The two solid lines (designated "constrained" in the lower right corner of the Figure) illustrate the effect of forcing the lag and gap regressions to be parallel.

The results of the probit analyses for the San Francisco studies in which lags are considered the stimulus appear in Table 1. The $\chi^2$ (chi-squared) statistic with $N_0 - 2$ degrees of freedom provides a test of the linearity of the transformed data. For all films except SF4-1 the computed $\chi^2$ statistics are small enough to be attributed to random variation. Examination of the data for SF4-1 reveals that at several short periods during the filming, traffic was congested at the ramp. As a consequence, in several instances ramp drivers were lined up on the ramp and forced to reject large lags. Such occurrences contribute inordinately large amounts to the $\chi^2$ statistic.

All the regression coefficients estimated are of the same order of magnitude. It is not evident, however, that all the $b_0$'s are equal. As an approximate test of the equality of the $b_0$'s a $\chi^2$ statistic with 7 degrees of freedom may be calculated as

$$\sum \frac{(S \text{wx}_o y_o)^2}{S \text{wx}_o^2} - \frac{(\Sigma \text{Sw}_x y_o)^2}{\Sigma \text{Sw}_x^2} = 27.17$$

which is clearly significant, indicating that the $b_0$'s are not all equal. Similar tests may be made on the $\chi^2$ for the equality of the $b_0$'s obtained from two films on the same ramp. These tests indicate that the $b_0$'s do not differ between films from the same ramp. Taken together, these results indicate that the regression coefficient is constant for a given ramp and some range of traffic conditions but that the $b_0$'s may
TABLE 1

RESULTS OF PROBIT ANALYSES ON EIGHT FILMS

Stimulus: $x_0$

Line Fitted: $Y = a_0 + b_0 x_0$

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td>196</td>
<td>197</td>
<td>291</td>
<td>311</td>
<td>328</td>
<td>164</td>
<td>159</td>
<td>179</td>
</tr>
<tr>
<td>$Sw$</td>
<td>76.949</td>
<td>78.517</td>
<td>76.961</td>
<td>116.140</td>
<td>141.424</td>
<td>41.048</td>
<td>59.216</td>
<td>57.774</td>
</tr>
<tr>
<td>$\bar{x}_0$</td>
<td>0.112</td>
<td>0.049</td>
<td>0.220</td>
<td>0.268</td>
<td>0.274</td>
<td>0.105</td>
<td>0.067</td>
<td>0.056</td>
</tr>
<tr>
<td>$\bar{y}_0$</td>
<td>0.762</td>
<td>0.686</td>
<td>0.846</td>
<td>0.723</td>
<td>0.345</td>
<td>0.530</td>
<td>0.708</td>
<td>0.461</td>
</tr>
<tr>
<td>$Swy_2$</td>
<td>189.883</td>
<td>193.564</td>
<td>239.783</td>
<td>354.345</td>
<td>511.978</td>
<td>182.492</td>
<td>161.794</td>
<td>193.001</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>160.78</td>
<td>158.582</td>
<td>181.065</td>
<td>294.534</td>
<td>422.694</td>
<td>140.881</td>
<td>131.593</td>
<td>142.136</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.609</td>
<td>0.608</td>
<td>0.309</td>
<td>0.267</td>
<td>-0.281</td>
<td>0.172</td>
<td>0.585</td>
<td>0.312</td>
</tr>
<tr>
<td>$b_0$</td>
<td>1.366</td>
<td>1.571</td>
<td>2.441</td>
<td>1.700</td>
<td>2.287</td>
<td>3.421</td>
<td>1.859</td>
<td>2.663</td>
</tr>
<tr>
<td>$m$</td>
<td>0.358</td>
<td>0.410</td>
<td>0.747</td>
<td>0.697</td>
<td>1.327</td>
<td>0.891</td>
<td>0.485</td>
<td>0.764</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.150</td>
<td>0.218</td>
<td>0.519</td>
<td>0.451</td>
<td>1.085</td>
<td>0.670</td>
<td>0.261</td>
<td>0.570</td>
</tr>
<tr>
<td>$m_u$</td>
<td>0.578</td>
<td>0.602</td>
<td>0.962</td>
<td>0.941</td>
<td>1.573</td>
<td>1.100</td>
<td>0.698</td>
<td>0.959</td>
</tr>
</tbody>
</table>
differ among ramps.

The results of probit analyses in which the first gap is considered the stimulus are shown in Table 2. The only large departure from linearity occurs in SF7-1 (see Figure 7). Examination of the data shows that this heterogeneity is attributable to several short periods during which cars were lined up on the ramp. In several instances ramp drivers rejected quite large gaps. The contribution to the $\chi^2$ from these instances is large. In each instance because the ramp driver was queued for the gap, ramp velocity was very slow compared to the speed at which freeway vehicles were traveling. As will be seen later adjustment for ramp velocity accounts for much of this apparent heterogeneity.

In general, the $b_1$'s are less than $b_0$'s. This is reasonable insofar as ramp drivers are less sensitive to small differences in gap size than to small differences in lag size. However, it should be noted that inclusion of queued ramp vehicles in the analysis for gaps may cause the regression coefficient to be underestimated. A case in point is that of SF8-1 in which the relatively small $b_1$ may be accounted for in part by the fact that there were several occasions in which a large gap was available to several cars of which the last rejected the gap. This emphasizes the importance of separating periods of stable flow from periods of congested flow in the analysis.

The last three rows of Tables 1 and 2 give the fifty percent point of the tolerance (median critical lag in Table 1 and median critical gap in Table 2), the lower 95% confidence limit (next to last row), and the upper 95% confidence limit (last row) for each of the San Francisco films. Again the results indicate differences among ramps but little difference, with the possible exception of SF8, between films of the same ramp. The 95% confidence limits for gap acceptance are illustrated in Figure 8.

Since a lag is a fraction of a gap by definition, one might expect that for any response (percent acceptance) the gap size producing that response should be a constant factor times the corresponding lag size. That is the probit lines should be parallel. To check this and to estimate the ratio of lag size to gap size (relative potency in the probit jargon) probit analyses were performed in which the two lines were constrained to be parallel.

The results of probit analyses in which the lines for lags and gaps are constrained to be parallel appear in Table 3. $\chi^2_{(LIN)}$ has $N_0 + N_1 - 3$ degrees of freedom and $\chi^2_{(PAR)}$ for tests of parallelism of
GAP ACCEPTANCE WITH CONFIDENCE LIMITS

FIGURE 8
TABLE 2
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS

Stimulus: $x_1$
Line Fitted: $y = a_1 + b_1 x_1$

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>196</td>
<td>197</td>
<td>291</td>
<td>311</td>
<td>328</td>
<td>164</td>
<td>159</td>
<td>179</td>
</tr>
<tr>
<td>$Sw$</td>
<td>80.146</td>
<td>79.427</td>
<td>109.890</td>
<td>141.276</td>
<td>181.660</td>
<td>75.080</td>
<td>76.572</td>
<td>86.609</td>
</tr>
<tr>
<td>$\bar{x}_1$</td>
<td>0.604</td>
<td>0.545</td>
<td>0.795</td>
<td>0.780</td>
<td>0.731</td>
<td>0.652</td>
<td>0.667</td>
<td>0.608</td>
</tr>
<tr>
<td>$\bar{y}_1$</td>
<td>0.749</td>
<td>0.665</td>
<td>0.988</td>
<td>0.811</td>
<td>0.387</td>
<td>0.747</td>
<td>0.826</td>
<td>0.579</td>
</tr>
<tr>
<td>$x$</td>
<td>198.931</td>
<td>199.508</td>
<td>317.135</td>
<td>337.973</td>
<td>365.436</td>
<td>210.780</td>
<td>165.145</td>
<td>195.213</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.396</td>
<td>-0.665</td>
<td>-0.495</td>
<td>-0.158</td>
<td>-0.684</td>
<td>-0.236</td>
<td>-0.387</td>
<td>-0.520</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.894</td>
<td>2.441</td>
<td>1.865</td>
<td>1.242</td>
<td>1.466</td>
<td>1.509</td>
<td>0.658</td>
<td>1.808</td>
</tr>
<tr>
<td>$m$</td>
<td>1.618</td>
<td>1.873</td>
<td>1.843</td>
<td>1.340</td>
<td>2.927</td>
<td>1.434</td>
<td>0.258</td>
<td>1.939</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.768</td>
<td>1.222</td>
<td>0.793</td>
<td>0.434</td>
<td>1.954</td>
<td>0.430</td>
<td>0.0</td>
<td>1.049</td>
</tr>
<tr>
<td>$m_{uu}$</td>
<td>2.306</td>
<td>2.398</td>
<td>2.742</td>
<td>2.234</td>
<td>3.781</td>
<td>2.308</td>
<td>1.279</td>
<td>2.665</td>
</tr>
</tbody>
</table>
the regression lines has one degree of freedom. The only significantly large $\chi^2_{\text{LIN}}$ is the one for SF7-1. This is probably due to the heterogeneity of the gap data as discussed earlier. There are three significant $\chi^2_{\text{PAR}}$ statistics for parallelism. Each comes from one of the troublesome films discussed earlier. It is thought that inasmuch as the departure from the model in these three cases are explicable in terms of gross freeway conditions unaccounted for in the present analyses, there need be no serious doubt as to the validity of the model. The analyses of Table 3 indicate that it is not wholly unreasonable to assume that $b_0 = b_1 = b$ for each ramp, though the b's may differ among ramps. The "constrained" regression lines appear as the solid lines on the graphs in Figure 7.

In conjunction with checking the parallelism of the two probit lines, the primary purpose of the analyses in Table 3 is to estimate the "relative potencies" $R_{0, 1}$ of lags to gaps: that is, to estimate the effectiveness of lags relative to gaps in inducing drivers to merge. This is estimated as the antilogarithm of the horizontal distance between the two parallel probit lines with $R$ being greater than unity in the case lying for lags to the left of the probit line for gaps. Thus a value of $R = 2$ implies that in order to induce the same percentage of response as a particular lag, the gap must be twice the size of the lag. It is seen that the estimates of $R$ in Table 3 the 3rd row from the bottom differ little among ramps indicating that the ratio of median critical lag to median critical gap is the same for all the San Francisco ramps.

Graphs of the lag and gap acceptance regressions for the remaining ramps, similar to Figure 7, appear in Appendix D. The graphs depicting confidence limits on gap acceptance for the remaining ramps, similar to Figure 8, appear in Appendix E.

Multiple Entries

A multiple entry occurs when two or more drivers accept the same gap. Three probit lines may be determined considering $r_i$ the responses (Appendix C) and $x_i$ the stimuli. The three lines are first fitted separately as:

$$Y_i = a_i + b_i x_i \quad i = 1, 2, 3.$$  

With these lines, it is possible to estimate for any given gap size the probability that one, two or three cars will accept that available gap.

Tables 4, 5 and 6 show the results of analyses for multiple entries.
TABLE 3
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS
Probit Lines for $x_0$ and $x_1$ Are Constrained to be Parallel:

\[ y_0 = a_0 + b x_0 \]
\[ y_1 = a_1 + b x_1 \]

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0+N_1$</td>
<td>392</td>
<td>394</td>
<td>582</td>
<td>622</td>
<td>656</td>
<td>328</td>
<td>318</td>
<td>358</td>
</tr>
<tr>
<td>$\Sigma Sw$</td>
<td>157.644</td>
<td>159.234</td>
<td>187.849</td>
<td>258.376</td>
<td>325.463</td>
<td>122.809</td>
<td>139.584</td>
<td>145.759</td>
</tr>
<tr>
<td>$\bar{x}_0$</td>
<td>0.093</td>
<td>0.025</td>
<td>0.236</td>
<td>0.285</td>
<td>0.283</td>
<td>0.149</td>
<td>0.117</td>
<td>0.073</td>
</tr>
<tr>
<td>$\bar{x}_1$</td>
<td>0.629</td>
<td>0.573</td>
<td>0.780</td>
<td>0.765</td>
<td>0.719</td>
<td>0.620</td>
<td>0.625</td>
<td>0.590</td>
</tr>
<tr>
<td>$\bar{y}_0$</td>
<td>0.736</td>
<td>0.647</td>
<td>0.881</td>
<td>0.750</td>
<td>0.364</td>
<td>0.651</td>
<td>0.781</td>
<td>0.496</td>
</tr>
<tr>
<td>$\bar{y}_1$</td>
<td>0.790</td>
<td>0.722</td>
<td>0.957</td>
<td>0.789</td>
<td>0.366</td>
<td>0.674</td>
<td>0.776</td>
<td>0.540</td>
</tr>
<tr>
<td>$\Sigma Swx^2$</td>
<td>21.452</td>
<td>18.682</td>
<td>18.166</td>
<td>36.923</td>
<td>35.850</td>
<td>13.024</td>
<td>18.031</td>
<td>15.152</td>
</tr>
<tr>
<td>$\Sigma Swxy$</td>
<td>32.769</td>
<td>34.310</td>
<td>40.113</td>
<td>56.015</td>
<td>68.866</td>
<td>29.918</td>
<td>23.975</td>
<td>32.693</td>
</tr>
<tr>
<td>$\Sigma Swy^2$</td>
<td>384.251</td>
<td>388.192</td>
<td>588.583</td>
<td>699.474</td>
<td>818.528</td>
<td>565.707</td>
<td>359.750</td>
<td>393.329</td>
</tr>
<tr>
<td>$\chi^2_{par}$</td>
<td>333.048</td>
<td>322.534</td>
<td>498.631</td>
<td>612.642</td>
<td>680.223</td>
<td>487.053</td>
<td>321.442</td>
<td>311.280</td>
</tr>
<tr>
<td>$\chi^2_{par}$</td>
<td>1.150</td>
<td>2.646</td>
<td>1.378</td>
<td>1.850</td>
<td>6.015</td>
<td>9.332</td>
<td>6.428</td>
<td>2.610</td>
</tr>
<tr>
<td>$\chi^2_{par}$</td>
<td>0.595</td>
<td>0.601</td>
<td>0.359</td>
<td>0.317</td>
<td>-0.180</td>
<td>0.309</td>
<td>0.626</td>
<td>0.328</td>
</tr>
<tr>
<td>$b$</td>
<td>1.528</td>
<td>1.837</td>
<td>2.208</td>
<td>1.519</td>
<td>1.921</td>
<td>2.297</td>
<td>1.330</td>
<td>2.290</td>
</tr>
<tr>
<td>$R_1$</td>
<td>1.926</td>
<td>2.133</td>
<td>2.366</td>
<td>1.940</td>
<td>2.089</td>
<td>1.999</td>
<td>1.762</td>
<td>2.238</td>
</tr>
</tbody>
</table>
The significantly large $\chi^2$ statistic for SF7-1 and the small $b_1$ for SF7-1 and the small $b_1$ for SF8-1 are explicable as before. For each ramp there is a tendency for the $b$'s to become successively larger: i.e., $b_1 < b_2 < b_3$ (see Figure 9). As the sensitivity of such platoons of cars to differences in gap size is affected by the decision of the last car in the platoon, this trend indicates the reasonable conclusion that the $i + 1$-th car in line for a given gap is more sensitive to differences in gap size than the $i$-th car. Furthermore, these analyses do not take account of ramp speed: it is expected that the $i + 1$-th vehicle in line must slow down more than the $i$-th vehicle in line. Thus barring any interaction of ramp speed and gap size, the probit line for three or more cars per gap would be shifted to the right of the probit line for two or more, and the line for two or more to the right of one or more. This would accentuate the shifting of the lines due to the simple fact that it takes a bigger gap for two cars than for one and for three than for two.

The 50% point and its 95% confidence limits are estimated and appear in the last three rows of Tables 4-6. In Table 4 the estimate of $m$ for SF8-1 is very small due to the position and very small slope of the probit line. The explanation for this is as presented earlier. The upper confidence limit was too large to fit into the space provided for it.

In Table 5 the $\chi^2$ for linearity for SF1-3 is significantly large. As in other such cases this statistic is inordinately inflated by contributions from a few cases in which less than 2 cars rejected large gaps. The $\chi^2$ statistic for SF7-1 again indicates misbehavior, the reasons being discussed earlier. Of special noteworthiness is the fact that the estimates for SF8-1 are much more reasonable than in Table 5. The elimination of the cases in which the gap was available to only one car evidently eliminates the heterogeneity in the data.

In Table 6 are the results of probit analyses for three or more cars per gap. The results for SF1-3 are typical of a situation encountered frequently when the gaps are not widely dispersed: the iterative procedure goes through more iterations than usual before converging, the $\chi^2$ statistic is very small and $b$ is very large.

Table 7 and Figure 10 shows the results of probit analyses in which the probit lines for one, two and three cars per gap are constrained to be parallel under the assumption that single, double, and triple entries are equally sensitive to differences in gap size. The $\chi^2_{\text{LIN}}$ for linearity has $N_1 + N_2 + N_3 - 4$ degrees of freedom and the $\chi^2_{\text{PAR}}$ for parallelism has two degrees of freedom. The $\chi^2_{\text{LIN}}$ statistic for SF1-1 is clearly
TABLE 4
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS
Multiple Entries: One or More Cars per Gap

\[ y_1 = a_1 + b_1 x_1 \]

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 )</td>
<td>120</td>
<td>129</td>
<td>131</td>
<td>146</td>
<td>176</td>
<td>104</td>
<td>106</td>
<td>114</td>
</tr>
<tr>
<td>( S_{w1} )</td>
<td>52.986</td>
<td>50.771</td>
<td>52.742</td>
<td>78.079</td>
<td>101.772</td>
<td>43.244</td>
<td>49.389</td>
<td>47.045</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.501</td>
<td>0.447</td>
<td>0.663</td>
<td>0.657</td>
<td>0.605</td>
<td>0.526</td>
<td>0.569</td>
<td>0.473</td>
</tr>
<tr>
<td>( y_1 )</td>
<td>0.696</td>
<td>0.573</td>
<td>0.782</td>
<td>0.586</td>
<td>0.269</td>
<td>0.707</td>
<td>0.869</td>
<td>0.603</td>
</tr>
<tr>
<td>( S_{wx1} )</td>
<td>2.687</td>
<td>2.175</td>
<td>2.817</td>
<td>7.480</td>
<td>8.053</td>
<td>3.521</td>
<td>3.899</td>
<td>2.645</td>
</tr>
<tr>
<td>( S_{wxy1} )</td>
<td>5.981</td>
<td>7.276</td>
<td>6.979</td>
<td>7.576</td>
<td>11.633</td>
<td>7.653</td>
<td>3.070</td>
<td>7.592</td>
</tr>
<tr>
<td>( S_{w2} )</td>
<td>138.611</td>
<td>129.110</td>
<td>144.049</td>
<td>153.215</td>
<td>198.445</td>
<td>152.464</td>
<td>112.043</td>
<td>128.414</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>125.298</td>
<td>104.770</td>
<td>126.759</td>
<td>145.542</td>
<td>181.641</td>
<td>135.830</td>
<td>108.626</td>
<td>105.623</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>-0.419</td>
<td>-0.922</td>
<td>-0.861</td>
<td>-0.080</td>
<td>-0.605</td>
<td>-0.437</td>
<td>0.421</td>
<td>-0.755</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>2.226</td>
<td>3.346</td>
<td>2.478</td>
<td>1.013</td>
<td>1.445</td>
<td>2.174</td>
<td>0.787</td>
<td>2.870</td>
</tr>
<tr>
<td>( m )</td>
<td>1.543</td>
<td>1.886</td>
<td>2.226</td>
<td>1.199</td>
<td>2.624</td>
<td>1.588</td>
<td>0.292</td>
<td>1.832</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>0.610</td>
<td>1.327</td>
<td>1.071</td>
<td>0.042</td>
<td>1.486</td>
<td>0.703</td>
<td>0</td>
<td>1.161</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>2.177</td>
<td>2.312</td>
<td>3.056</td>
<td>2.371</td>
<td>3.594</td>
<td>2.289</td>
<td>--</td>
<td>2.351</td>
</tr>
</tbody>
</table>
TABLE 5
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS
Multiple Entries: Two or More Cars per Gap

\[ Y_2 = a_2 + b_2 x_1 \]

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_2 )</td>
<td>63</td>
<td>71</td>
<td>89</td>
<td>107</td>
<td>139</td>
<td>64</td>
<td>53</td>
<td>70</td>
</tr>
<tr>
<td>( S_{w_2} )</td>
<td>23.662</td>
<td>27.351</td>
<td>42.237</td>
<td>54.298</td>
<td>69.988</td>
<td>26.360</td>
<td>29.025</td>
<td>32.147</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.675</td>
<td>0.601</td>
<td>0.752</td>
<td>0.760</td>
<td>0.707</td>
<td>0.667</td>
<td>0.701</td>
<td>0.677</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>-0.104</td>
<td>-0.064</td>
<td>0.287</td>
<td>0.108</td>
<td>-0.253</td>
<td>0.118</td>
<td>-0.034</td>
<td>-0.190</td>
</tr>
<tr>
<td>( S_{w_{x_1}^2} )</td>
<td>0.975</td>
<td>1.110</td>
<td>2.240</td>
<td>3.965</td>
<td>4.720</td>
<td>1.400</td>
<td>2.513</td>
<td>2.031</td>
</tr>
<tr>
<td>( S_{w_{x_2}^2} )</td>
<td>4.394</td>
<td>4.833</td>
<td>7.167</td>
<td>10.519</td>
<td>12.093</td>
<td>5.402</td>
<td>5.108</td>
<td>6.280</td>
</tr>
<tr>
<td>( S_{w_{y_2}^2} )</td>
<td>80.073</td>
<td>159.604</td>
<td>112.572</td>
<td>130.943</td>
<td>160.872</td>
<td>103.515</td>
<td>65.593</td>
<td>79.628</td>
</tr>
<tr>
<td>( x )</td>
<td>60.271</td>
<td>138.561</td>
<td>89.641</td>
<td>103.037</td>
<td>129.889</td>
<td>82.671</td>
<td>55.211</td>
<td>60.210</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>4.507</td>
<td>4.356</td>
<td>3.199</td>
<td>2.653</td>
<td>2.562</td>
<td>3.857</td>
<td>2.033</td>
<td>3.093</td>
</tr>
<tr>
<td>( m )</td>
<td>4.987</td>
<td>4.131</td>
<td>4.600</td>
<td>5.244</td>
<td>6.394</td>
<td>4.329</td>
<td>5.219</td>
<td>5.480</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>4.008</td>
<td>3.341</td>
<td>3.407</td>
<td>4.016</td>
<td>5.180</td>
<td>3.302</td>
<td>3.173</td>
<td>4.213</td>
</tr>
<tr>
<td>( m_u )</td>
<td>6.364</td>
<td>5.186</td>
<td>5.715</td>
<td>6.646</td>
<td>8.419</td>
<td>5.498</td>
<td>8.986</td>
<td>7.643</td>
</tr>
</tbody>
</table>
TABLE 6
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS
Multiple Entries: Three or More Cars per Gap
\[ Y_3 = a_3 + b_3 x_1 \]

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_3</td>
<td>51</td>
<td>52</td>
<td>72</td>
<td>90</td>
<td>120</td>
<td>40</td>
<td>37</td>
<td>53</td>
</tr>
<tr>
<td>x_1</td>
<td>0.883</td>
<td>0.868</td>
<td>0.833</td>
<td>0.876</td>
<td>0.908</td>
<td>0.920</td>
<td>0.869</td>
<td>0.871</td>
</tr>
<tr>
<td>y_3</td>
<td>-0.210</td>
<td>-0.163</td>
<td>0.057</td>
<td>-0.047</td>
<td>-0.704</td>
<td>-0.325</td>
<td>-0.665</td>
<td>-0.550</td>
</tr>
<tr>
<td>Swx_2</td>
<td>0.297</td>
<td>0.015</td>
<td>0.709</td>
<td>1.842</td>
<td>1.609</td>
<td>0.471</td>
<td>0.979</td>
<td>0.283</td>
</tr>
<tr>
<td>Swx_1 y_3</td>
<td>1.853</td>
<td>0.221</td>
<td>4.071</td>
<td>7.126</td>
<td>6.110</td>
<td>2.065</td>
<td>2.685</td>
<td>1.561</td>
</tr>
</tbody>
</table>
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 9
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES (CONSTRAINED OR PARALLEL)

FIGURE 10
significant even though none of the \(^2\)\text{LIN} statistics for the analyses on SF1-1 in Tables 4, 5 or 6 were significant. This phenomenon results from the different weights for each observation in the present analysis as compared to the analyses of Tables 4-6. Thus, if the b's are calculated separately using the weights of the present analysis, \(b_1 = 1.811\), \(b_2 = 4.278\) and \(b_3 = 4.825\) result.

Because of the underestimation of \(b_1\) in the present analysis and the large number of vehicles in the "one or more" group, the contribution to the \(^2\)\text{LIN} of 278 from that group is great. The \(^2\)\text{PAR} for SF1-1 is also significantly large though not extremely so. SF3-1, SF3-2, and SF4-1 show significantly large \(^2\)\text{PAR} statistics. For each ramp, with the exception of SF7-1, the trend \(b_1 < b_2 < b_3\) is quite pronounced and the \(^2\)\text{PAR} statistics fairly large (all are significant at the 95% level). This fact indicates that the model considering the lines parallel is statistically inadequate. It may be concluded that double entries are more sensitive than singles, and triples than doubles, to differences in gap size. Even so, the analysis may be of use in assessing the sizes of equally effective gaps for two or more \((R_1, 2)\) and three or more \((R_{1, 3})\) vehicles relative to one or more. These estimates and their 95% confidence limits are tabulated in Table 7, as are the estimated 50% gap points for one or more \((m_1)\), two or more \((m_2)\) and three or more \((m_3)\) vehicles.

Graphs of gap acceptance for multiple vehicle merges similar to Figures 9 and 10 appear in Appendix F for the unconstrained and Appendix G for the constrained for the remaining ramps.
### TABLE 7

RESULTS OF PROBIT ANALYSES ON EIGHT FILMS

Multiple Entries: Parallel Analyses

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_1+N_2+N_3)</td>
<td>234</td>
<td>252</td>
<td>292</td>
<td>343</td>
<td>435</td>
<td>208</td>
<td>196</td>
<td>237</td>
</tr>
<tr>
<td>(S_w)</td>
<td>92.158</td>
<td>85.845</td>
<td>124.218</td>
<td>177.243</td>
<td>209.401</td>
<td>81.677</td>
<td>93.190</td>
<td>91.297</td>
</tr>
<tr>
<td>(1) (\bar{x}_1)</td>
<td>0.468</td>
<td>0.432</td>
<td>0.636</td>
<td>0.614</td>
<td>0.594</td>
<td>0.496</td>
<td>0.530</td>
<td>0.464</td>
</tr>
<tr>
<td>(2) (\bar{y}_1)</td>
<td>0.674</td>
<td>0.601</td>
<td>0.750</td>
<td>0.758</td>
<td>0.700</td>
<td>0.660</td>
<td>0.697</td>
<td>0.679</td>
</tr>
<tr>
<td>(3) (\bar{y}_1)</td>
<td>0.819</td>
<td>0.798</td>
<td>0.830</td>
<td>0.824</td>
<td>0.824</td>
<td>0.845</td>
<td>0.795</td>
<td>0.812</td>
</tr>
<tr>
<td>(\bar{y}_2)</td>
<td>0.594</td>
<td>0.516</td>
<td>0.703</td>
<td>0.513</td>
<td>0.240</td>
<td>0.623</td>
<td>0.815</td>
<td>0.575</td>
</tr>
<tr>
<td>(\bar{y}_3)</td>
<td>-0.108</td>
<td>-0.071</td>
<td>0.281</td>
<td>0.105</td>
<td>-0.264</td>
<td>0.113</td>
<td>-0.032</td>
<td>-0.185</td>
</tr>
<tr>
<td>(\bar{y}_4)</td>
<td>-0.498</td>
<td>-0.654</td>
<td>0.060</td>
<td>-0.189</td>
<td>-0.932</td>
<td>-0.577</td>
<td>-0.834</td>
<td>-0.781</td>
</tr>
<tr>
<td>(S_w \cdot x_1^2)</td>
<td>4.412</td>
<td>3.476</td>
<td>6.024</td>
<td>14.645</td>
<td>15.177</td>
<td>5.877</td>
<td>7.926</td>
<td>5.125</td>
</tr>
<tr>
<td>(S_w \cdot x_1 y)</td>
<td>398.657</td>
<td>305.305</td>
<td>415.890</td>
<td>415.154</td>
<td>487.346</td>
<td>418.959</td>
<td>234.981</td>
<td>251.426</td>
</tr>
<tr>
<td>(S_w \cdot x_2^2)</td>
<td>15.180</td>
<td>14.427</td>
<td>20.213</td>
<td>30.123</td>
<td>32.574</td>
<td>17.184</td>
<td>12.118</td>
<td>16.476</td>
</tr>
<tr>
<td>(S_w \cdot x_2 y)</td>
<td>338.596</td>
<td>241.512</td>
<td>317.583</td>
<td>337.517</td>
<td>407.910</td>
<td>364.094</td>
<td>211.783</td>
<td>196.815</td>
</tr>
<tr>
<td>(S_w \cdot x_3^2)</td>
<td>7.834</td>
<td>3.925</td>
<td>6.179</td>
<td>15.677</td>
<td>9.528</td>
<td>4.617</td>
<td>4.671</td>
<td>1.645</td>
</tr>
<tr>
<td>(S_w \cdot x_3 y)</td>
<td>1.016</td>
<td>-1.275</td>
<td>-1.431</td>
<td>-0.750</td>
<td>-1.034</td>
<td>-0.827</td>
<td>0.004</td>
<td>-0.917</td>
</tr>
<tr>
<td>(a_1)</td>
<td>-2.427</td>
<td>-2.564</td>
<td>-2.237</td>
<td>-1.454</td>
<td>-1.767</td>
<td>-1.818</td>
<td>-1.097</td>
<td>-2.368</td>
</tr>
<tr>
<td>(b)</td>
<td>3.441</td>
<td>4.150</td>
<td>3.356</td>
<td>2.057</td>
<td>2.146</td>
<td>2.924</td>
<td>1.529</td>
<td>3.215</td>
</tr>
<tr>
<td>(R_{1,2})</td>
<td>2.571</td>
<td>2.045</td>
<td>1.738</td>
<td>2.199</td>
<td>2.194</td>
<td>2.183</td>
<td>5.253</td>
<td>2.828</td>
</tr>
<tr>
<td>(R_{1})</td>
<td>1.874</td>
<td>1.576</td>
<td>1.306</td>
<td>1.492</td>
<td>1.578</td>
<td>1.503</td>
<td>2.538</td>
<td>2.031</td>
</tr>
<tr>
<td>(R_{3})</td>
<td>4.455</td>
<td>4.451</td>
<td>2.430</td>
<td>3.556</td>
<td>5.967</td>
<td>5.745</td>
<td>22.026</td>
<td>5.881</td>
</tr>
<tr>
<td>(R_{L})</td>
<td>3.177</td>
<td>3.086</td>
<td>1.800</td>
<td>2.335</td>
<td>3.848</td>
<td>3.425</td>
<td>7.474</td>
<td>3.768</td>
</tr>
<tr>
<td>(R_{U})</td>
<td>7.660</td>
<td>7.019</td>
<td>3.461</td>
<td>6.008</td>
<td>10.709</td>
<td>11.269</td>
<td>238.072</td>
<td>10.685</td>
</tr>
<tr>
<td>(m_1)</td>
<td>1.973</td>
<td>2.028</td>
<td>2.670</td>
<td>2.316</td>
<td>3.033</td>
<td>1.917</td>
<td>0.994</td>
<td>1.928</td>
</tr>
<tr>
<td>(m_2)</td>
<td>5.074</td>
<td>4.148</td>
<td>4.641</td>
<td>5.092</td>
<td>6.655</td>
<td>4.186</td>
<td>5.220</td>
<td>5.453</td>
</tr>
</tbody>
</table>
The Ideal Merge

In the first part of this report, it was suggested that a merge must be qualified according to the terminology illustrated in Figure 2. Thus, the most desirable type of "gap" merge would be both "optional" and "ideal" in that it would not be made because the ramp driver had run out of acceleration lane nor would it cause turbulence in the freeway stream. The requirements of such a merge serve to document the interaction of the basic traffic elements—the driver and the vehicle, and the basic traffic characteristics—headways and vehicular speeds.

If it is assumed that the average driver's normal acceleration of the merging vehicle may be represented by the following differential equation

\[
\frac{du}{dt} = a - bu
\]  

(56)

where \( u \) is the speed of the merging vehicle, \( t \) is time, and \( a \) and \( b \) are constants. If the merging vehicle is moving at a speed \( u_r \) at the beginning of the merge, then the limits of integration for (56) are

\[
\int_{u_r}^{u} \frac{-bdu}{a-bu} = \int_{0}^{t} dt
\]

and the speed-time relationship is

\[
\ln\left(\frac{a - bu}{a - bu_r}\right) \bigg|_{u_r}^{u} = t
\]

\[
\frac{a - bu}{a - bu_r} = e^{-bt}
\]

\[
u = \frac{a}{b} \left(1 - e^{-bt}\right) + u_r e^{-bt}
\]  

(57)

Since \( u = dx/dt \), integration of (57) provides the equation of the time-space curve.
\[ x = \frac{a}{b} t - \frac{a}{b^2} (1 - e^{-bt}) + \frac{u}{b} (1 - e^{bt}) \]  

(58)

Substitution of (57) in (56) gives the acceleration-time relationship for the merging maneuver:

\[ \frac{du}{dt} = (a - bu_r) e^{-bt} \]  

(59)

The units of the constants \( a \) and \( b^{-1} \) are those of acceleration and time respectively, where \( a \) is the maximum acceleration and \( (a/b) \) is the free speed in the merging area. The forms of Equations 56 to 59 are illustrated in Figure 11.

The time-space relationship of Figure 11 has been reproduced in Figure 12 along with the procedure for determining the theoretical minimum ideal gap for merging. Such a gap is made up of three time intervals: (1) a safe time headway between the merging vehicle and the freeway vehicle ahead \( T_r \), (2) the time lost accelerating during the merging maneuver, \( T_L \), (3) a safe time headway between the second freeway vehicle and the merging vehicle \( T_f \). The safe headway referred to is that headway between two vehicles in a lane which will allow the following vehicle to stop safely even if the vehicle in front makes an emergency stop. If reaction times \( \tau \), braking capabilities and speeds \( u \) are assumed to be equal, then the safe headway is \( (L/u) + \tau \) where \( L \) is the length of the vehicle in front.

The time necessary for the merging ramp vehicle to accelerate from a speed \( u_r \) at the beginning of the merge to attain the speed of the freeway traffic \( u \) may be obtained by solving Equation 57 for time.

\[ T_2 = - \frac{1}{b} \ln \left( \frac{a - bu}{a - bu_r} \right) \]  

(60)

Subtracting the travel time \( T_1 \) to travel the same distance covered during merging, only at a constant speed \( u \), from (60) gives the time lost during merging \( T_L \). The theoretical minimum ideal gap for merging \( (T_r + T_f + T_L) \) is

\[ T = \frac{L_f + L_r}{u} + 2\tau + \frac{u + u_r}{bu} + \frac{(a/b) - u}{bu} \ln \left( \frac{a - bu}{a - bu_r} \right) \]  

(61)

in which \( L_f \) and \( L_r \) are the lengths of the freeway and ramp vehicles.
THEORETICAL SPEED-ACCELERATION RELATIONSHIP FOR A MERGING VEHICLE

Figure 11
Figure 12

THEORETICAL MINIMUM IDEAL GAP FOR MERGING

$T_r =$ SAFE TIME HEADWAY BETWEEN RAMP AND LEAD VEHICLES

$T_l =$ TIME LOST ACCELERATING DURING MERGING

$T_f =$ SAFE TIME HEADWAY BETWEEN LAG AND RAMP VEHICLES

$T_i =$ MINIMUM IDEAL GAP
It is apparent from (61) that a merging truck would require a larger gap than a merging passenger car by virtue of its longer length, $L_r$, and its reduced accelerating capabilities, $a$. Similarly, if the first of the two freeway vehicles is a truck, the merging vehicle requires a larger gap because of an increased $L_f$ in (61).

In order to determine representative values of minimum safe gaps, the parameters $a$ and $b$ in equation (61) must be estimated. Haight suggests that some information on the parameters could be obtained from drag races which are now widely held. Knox used the average result of road tests obtaining values of $a = 4.8$ mph/sec. and $a/b = 80$ mph for Australian conditions.

**Speed of the Merging Vehicle**

It is apparent from the previous section that the ramp drivers' problem in executing the merging maneuver is more than one of simply evaluating successive time headways in the freeway traffic stream until he finds a large enough gap. For example, Equation 61 shows that the ideal gap is based on the ramp driver's reaction time $\tau$; the vehicle characteristics $L_f$, $L_r$, $a$, and $b$; and the speeds of the mainstream vehicles and ramp vehicle $u$ and $u_r$. The influence of speeds on merging should come as no surprise; we know that, for example, under conditions of forced flow on the freeway that comparatively large time headways are rejected. Theoretically, infinitely large time headways would necessarily be rejected as the concentration and movement ceased.

In delineating the effect of speeds on merging, consider a very simple model consisting of two vehicles A and B separated by a space headway $s$ traveling at a constant velocity $u$ on the outside lane of the freeway, and a vehicle C traveling at speed $u_r$ on a corner intersecting the freeway at an angle $\delta$. It follows that the gap between A and B is

$$t = \frac{s}{u} \quad (62)$$

However, since the driver in vehicle C is moving he views the time gap between A and B as

$$t' = \frac{s}{u - u_r \cos \delta} \quad (63)$$

where $u_r \cos \delta$ is vehicle C's speed component along the freeway. Solving for $s$ in (63) and substituting in (62) yields
\[
\begin{align*}
\frac{u-u_r \cos \delta}{t'} &= \left(\frac{r}{u}\right) t \\
\end{align*}
\]

If \(t' > T'\) where \(T'\) is a constant, the gap will be accepted. Therefore, the minimum acceptable gap \(T\),

\[
\begin{align*}
T &= \left(\frac{u-u_r \cos \delta}{u}\right) T' \\
\end{align*}
\]

is clearly a function of the speed of the freeway traffic \(u\) and the speed of the ramp vehicle \(u_r\).

The effect of ramp geometry on merging operation (the subject of a companion report\textsuperscript{21}) is indicated in Equation 65. The equation suggests that the difference between traffic operation at an ordinary intersection and that at a freeway ramp terminal is primarily in the angle at which entering traffic and through traffic converge. For an intersection (\(\delta=90^\circ\)) one obtains \(T=T'\) from (65); for a lane change (\(\delta=0^\circ\))

\[
\begin{align*}
T &= \left(\frac{u-u_r}{u}\right) T' \\
\end{align*}
\]

Whereas, for an intersection, gap acceptance is independent of the speed of the merging vehicle, the speed of a vehicle merging from an entrance ramp has a profound effect on gap acceptance. This is illustrated in the graphs in Figure 13 for the California ramps. The dashed line pertains to ramp vehicles with speeds at the beginning of the merge greater than 20 mph; the solid line to ramp vehicles with speeds at the beginning of the merge less than 20 mph. For most of the California ramps there were virtually no slow moving ramp vehicles hence only the \(>20\) mph line appears. However, for the Ashby, Pleasant Hill and Sacramento ramps the effect of the speed of the merging vehicle \(u_r\) in accordance with the model exemplified by Equation 65 is evidenced. For the higher speed a much smaller gap was needed.

Graphs of gap acceptance for fast and slow merging vehicles for the remaining ramps studied appear in Appendix H.

**Angular Velocity Model**

Several researchers have hypothesized the use of angular velocity as the basis for the gap acceptance decision. The advantage of this
GAP ACCEPTANCE FOR FAST & SLOW MERGING VEHICLES

FIGURE 13
parameter is it takes into consideration the distance of the approaching freeway vehicle and its speed of approach.

The angular velocity model for gap acceptance is an application of the calculus of related rates. Referring to the moving coordinate system in Figure 14, we can write the equations for the relative speed of the freeway vehicle and merging vehicle and their space lag:

\[
\frac{dx}{dt} = u - u_r \tag{67}
\]

\[-x = w \cot \theta \tag{68}\]

Differentiating (68) with respect to time, equating it to (67) and solving for the angular velocity \( \theta \) yields

\[
\dot{\theta} = \frac{w}{s} (u - u_r) \tag{69}
\]

The basic aspect of the theory is that a ramp driver rejects a lag if he detects an angular velocity but accepts the lag if he cannot perceive any motion. Michaels and Weingarten\textsuperscript{22} utilize this criterion to develop the equation for the minimum acceptable gap \( T \).

\[
T = \frac{s}{u} \tag{70}
\]

\[
= \frac{w}{\dot{\theta} u} (u - u_r)^{1/2}
\]

Although the direct application of equation (70) to determine gap acceptance times depends on the evaluation of drivers' threshold of angular velocity, the role of speed in gap acceptance is again evidenced. The relative speed \((u - u_r)\) appearing in the numerator of (70) suggests that the lower the relative speed the smaller the gap needed. The speed of the mainstream traffic \( u \) in the denominator indicates an inverse relationship. For low speed freeway traffic, a larger gap is needed. Moreover, the fact that the latter variable is raised to a higher power than relative speed suggests it might have more effect on the gap acceptance performance.

Before the effect of speeds on gap acceptance is explored further, some aspects of a two variable probit analysis need to be discussed.
THEORETICAL MINIMUM ACCEPTABLE GAP FOR MERGING

Figure 14
Results of Two Variable Probit Analysis

It has been demonstrated in the single variable probit analysis how the acceptance curve (percent acceptance as a function of the single variable gap or lag size) may be transformed into a straight line. Just as a regression analysis may involve more than one independent variable, a probit analysis may be generalized to include more than one independent variable. Based on the theory of the preceding sections the two obvious independent variables are gap (or lag) and some speed parameter (u, u_r, or u-u_r). Figure 15 illustrates a hypothetical gap acceptance surface for a freeway speed u = 50 mph. A vertical plane parallel to the T axis yields a gap acceptance curve of the form shown Figure 6d and discussed in the single variable analysis. It should be apparent that the equation of the surface in Figure 15 would be difficult to write—the value of the parameters of the equation from the row data even more difficult to estimate.

A probit analysis for each ramp is presented in which a combination of x_0 and u_r is considered the stimulus and r the response. In each analysis there is obtained an estimate of a probit plane defined by Y = a + b_1x_0 + b_2 u_r. Thus for any combination of x_0 and u_r, it is possible to estimate Y and the corresponding percentage of all ramp drivers who would accept that combination. Or, going in reverse, it is possible to estimate for any given percentage acceptance the corresponding combinations of x_0 and u_r producing that percentage. The effect of the two variables probit analysis is to change an acceptance plane such as shown in Figure 16. Again, Figures 15 and 16 are meant to be illustrative rather than conclusive, the actual independent variables used being the lag x_0 and the speed of the ramp vehicle u_r.

Table 8 shows the results of analyses in which x and u_r together constitute the stimulus variable. The $\chi^2$ statistic with N-3 degrees of freedom provides a test of linearity. The only significant $\chi^2$ is that for SF4-1; again, the congested traffic conditions probably caused the apparent heterogeneity. Ramp SF7 shows the greatest effect of ramp speed on lag acceptance; this result supports the arguments proffered earlier explaining the apparent heterogeneity of data for this ramp. The last two rows of Table 8 give estimates of the 50% points for lags when u_r = $\bar{u}_r$ and u_r = 0.

The results of the analysis summarized in Table 8 are shown in Figure 17 for the California ramps. The three lines on each graph depict from left to right a speed at the start of the merge of 50 mph, the mean speed for the study period and 0 mph. Similar graphs for the remaining ramps appear in Appendix I.
GAP ACCEPTANCE SURFACE FOR FREEWAY SPEED, 
U = 50 MPH

Figure 15
Figure 16

ACCEPTANCE PLANE
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 17
The two variable probit analysis was repeated using the relative speed of the merging vehicle with respect to the freeway lag vehicle,

\[ Y = a + b_1 x_o + b_2 (u-u_r) \]

The results for the California ramps, shown in Figure 18, verify the models suggested by Equation 65 and 70. Similar graphs for the remaining ramps appear in Appendix J.

The coefficients for all two variable probit analyses are summarized in Appendix K.
TABLE 8
RESULTS OF PROBIT ANALYSES ON EIGHT FILMS

Two-Dose Analyses: \( Y = a + b_1 x_0 + b_2 V_r \)

<table>
<thead>
<tr>
<th>Film:</th>
<th>SF1-1</th>
<th>SF1-3</th>
<th>SF3-1</th>
<th>SF3-2</th>
<th>SF4-1</th>
<th>SF7-1</th>
<th>SF8-1</th>
<th>SF8-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>196</td>
<td>197</td>
<td>291</td>
<td>311</td>
<td>328</td>
<td>164</td>
<td>159</td>
<td>179</td>
</tr>
<tr>
<td>Sw</td>
<td>70.991</td>
<td>74.667</td>
<td>68.148</td>
<td>111.185</td>
<td>139.179</td>
<td>31.596</td>
<td>58.869</td>
<td>53.082</td>
</tr>
<tr>
<td>( x_0 )</td>
<td>0.098</td>
<td>0.048</td>
<td>0.203</td>
<td>0.270</td>
<td>0.275</td>
<td>0.097</td>
<td>0.064</td>
<td>0.056</td>
</tr>
<tr>
<td>( V_r )</td>
<td>26.110</td>
<td>26.382</td>
<td>37.184</td>
<td>34.210</td>
<td>30.343</td>
<td>37.753</td>
<td>31.205</td>
<td>32.708</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.711</td>
<td>0.672</td>
<td>0.722</td>
<td>0.723</td>
<td>0.348</td>
<td>0.378</td>
<td>0.702</td>
<td>0.436</td>
</tr>
<tr>
<td>SwV' ( V )</td>
<td>7550.095</td>
<td>7262.986</td>
<td>2713.574</td>
<td>10108.283</td>
<td>4280.659</td>
<td>767.460</td>
<td>3072.899</td>
<td>1512.316</td>
</tr>
<tr>
<td>Swx' ( V_r )</td>
<td>-114.085</td>
<td>-77.616</td>
<td>-63.768</td>
<td>-35.274</td>
<td>-8.788</td>
<td>-27.331</td>
<td>-29.845</td>
<td>-17.034</td>
</tr>
<tr>
<td>SwV' y ( V_r )</td>
<td>123.334</td>
<td>106.030</td>
<td>56.143</td>
<td>238.825</td>
<td>133.231</td>
<td>-3.576</td>
<td>-13.511</td>
<td>68.524</td>
</tr>
<tr>
<td>Swy ( V )</td>
<td>180.575</td>
<td>182.233</td>
<td>245.760</td>
<td>349.495</td>
<td>505.946</td>
<td>105.377</td>
<td>159.074</td>
<td>182.510</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>147.891</td>
<td>145.119</td>
<td>186.437</td>
<td>287.857</td>
<td>413.515</td>
<td>72.984</td>
<td>128.497</td>
<td>131.747</td>
</tr>
<tr>
<td>a</td>
<td>-0.498</td>
<td>-0.275</td>
<td>-3.063</td>
<td>-0.747</td>
<td>-1.362</td>
<td>-6.019</td>
<td>0.142</td>
<td>-2.236</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>1.588</td>
<td>1.705</td>
<td>2.800</td>
<td>1.700</td>
<td>2.268</td>
<td>4.562</td>
<td>1.902</td>
<td>2.805</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.040</td>
<td>0.033</td>
<td>0.086</td>
<td>0.030</td>
<td>0.036</td>
<td>0.158</td>
<td>0.014</td>
<td>0.077</td>
</tr>
<tr>
<td>( V_r = \bar{V}_r \cdot m )</td>
<td>0.447</td>
<td>0.450</td>
<td>0.882</td>
<td>0.699</td>
<td>1.323</td>
<td>1.032</td>
<td>0.495</td>
<td>0.795</td>
</tr>
<tr>
<td>( V_r = m )</td>
<td>2.058</td>
<td>1.451</td>
<td>12.419</td>
<td>2.751</td>
<td>3.985</td>
<td>20.875</td>
<td>0.842</td>
<td>6.270</td>
</tr>
</tbody>
</table>
EFFECT OF RELATIVE SPEED ON LAG ACCEPTANCE

FIGURE 18
SUMMARY

Theory

The announced objectives of this phase of the project research listed in the introduction will be discussed in conjunction with the findings.

The first objective of this particular study was the development of models and useful parameters for describing the freeway merging process. This theoretical development began with the derivation of the forms for the mean and variance of the delay to a ramp vehicle in position to merge in terms of the ramp driver's mean critical gap, freeway flow, and freeway gap availability expressed in terms of the appropriate Erlang distribution (Equations 27-34). This, a generalization of previous delay models, does not appear in the literature. The importance of this delay parameter lies in its applicability as a means to accurately forecast the capacity and operating conditions which can be expected for individual ramp-freeway junction designs. This application is the subject of a future report.

The remainder of the theoretical development concerns the treatment of the variability of critical gaps and gap acceptance among drivers. The significance of this portion of the investigation lies in the identification and application of the representative forms for both critical gap distributions and gap acceptance functions.

Characteristics

The emphasis in this report, consistent with specific objectives 2-5 in the introduction, is the collection and collation of gap acceptance characteristics. The fact that 32 ramps—chosen to reflect diverse operating, geometric, geographic and environmental conditions (See Appendix A)—were continuously filmed at 5 frames per second for an average of an hour, and that enough data was collected to run 1344 usable gap acceptance regressions serve to demonstrate not only the vast quantity of data involved, but the nature of the characteristics now available to interested researchers.

It is thought that the application of the individual record probit analysis has provided simple, statistically significant linear relations between gap acceptance and gap size. The idea of constraining pertinent regression lines to achieve parallelism affords the means of quantitatively comparing lags to gaps, single entry to multiple entry merges.
and fast to slow merging vehicles.

Specific objective (3) in the introduction needs some explanation. It is well known that the driver of a merging vehicle possesses a certain ability to adjust his merging environment. By speeding up or slowing down, he may adjust his position relative to that of a sequence of free-way vehicles, thereby enhancing his opportunity to merge. Indeed, in the computer output exemplified in Appendix B an appreciable percentage of drivers accepted a gap before the first available gap (denoted by a minus sign before the number of the gap accepted) simply by overtaking it. To account for this "dynamic" aspect of merging, lag acceptance characteristics at points along the acceleration lane were measured and summarized in the conventional graphical format used throughout this report. However, because of its applicability, these graphs are to be included in another project report establishing the effects of ramp and acceleration lane geometrics on merging operation.

Figure 13 and Appendix H suggested that not only might the percent acceptance be related to the logarithm of the time gap for fixed freeway and ramp vehicle speeds, but that this linearity might extend to the logarithm of these speeds for a fixed critical gap. Therefore, as a representation of the effect of time and speed on gap acceptance, a surface similar to the one shown in Figure 15 seemed appropriate. To fit such a surface (with the desired log normal curve in the percent acceptance-time plane) to real data, and then test it statistically, a two variable probit analysis was used. Under this transformation, the surface of Figure 15 becomes the plane of Figure 16. This model for the percent acceptance-time-speed data proved statistically reliable tending to establish that the logarithms of the critical gap and some function of speed (relative or absolute) required to give any percent acceptance are linearly related.

Application

The effect of outside freeway lane volumes on gap acceptance has not been discussed. In a study of lag acceptance at urban intersection, Raff wrote "it is safe to say that the main-street volume does not have an appreciable effect in the critical lag". Similarly for ramps, Wohl attributed a slightly smaller percent acceptance at higher freeway volumes to sampling rather than a difference in driver behavior.

The results of this investigation tend to support the conclusions of Raff and Wohl. The graphs for individual films taken at the same location illustrate the consistancy of the gap and lag acceptance curves
(See Figure 7 and Appendix D). However, since gap acceptance is a function of speed and speed is a function of volume, a relationship does exist. Because of the form of the speed-volume relationship (the classic parabola showing two speeds for any given volume), it is evident that the volume-gap acceptance relationship is a very complex one. It would suffice to say that gap acceptance is independent of volume for a given level of service (free, stable, unstable, or forced flow) and that changes in gap acceptance between different levels of service may be predicted from differences in speed rather than the service volumes. From a practical point of view, since a much larger gap is needed for a freeway volume of 1800 vph under a forced flow condition (20 mph) than under a stable flow condition (40 mph), positive means should be taken to prevent the freeway from becoming congested.

An important application of the results of this report lies in the field of simulation. Computer technologists have been quite active with several digital computer programs developed for simulating freeway and interchange operation. As a design and operational tool, present programs are not sufficiently validated to warrant confidence in their ability to predict behavior or needs at freeway interchanges. While development of simulator hardware and programs has proven feasible, lack of detailed criteria on traffic stream interaction has hampered progress. While some simplifications are necessary in simulation models, simplification necessitated because of lack of knowledge of the pertinent variables reduces the model's realism. Over-simplification of the merging logic is a case in point. For example, distributions for merging gap acceptance under different-geometric, traffic, and environmental conditions have been lacking.

The results of the probit analyses serve to present the pertinent gap acceptance variables in the form of distributions (the log-normal) and equations (the linear probit equation), making them extremely usable as inputs to digital simulation models. A sub-routine is being written to perform this.

As mentioned before, this Project is but the first phase of a four-year research program the Bureau of Public Roads is undertaking to (1) furnish more detailed information on the effect that geometric variables have on the merging of ramp traffic, (2) develop usable distributions of traffic variables for simulation programs, and (3) develop an optimum ramp metering and merging control system. The material presented in this report should aid materially in fulfilling these ultimate objectives.

The importance of preventing freeway congestion was alluded to
earlier in this Summary with reference to the volume-gap acceptance relationship. Of course, the concept of ramp control as a means of deferring congestion is now a reality. It only remains for these ramp control systems to be generalized to include merging control. The design of such a merging control system will depend on the proper interpretation of merging gap acceptance characteristics.
REFERENCES


5. Tanner, J. C., "The Delay to Pedestrians Crossing a Road," Biometrika, 38:3 and 4, December, 1951.


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APPENDICES
## APPENDIX A

### Summary of Geometrics of Ramp Study Locations

<table>
<thead>
<tr>
<th>Location</th>
<th>Ramp Name</th>
<th>Length and Angle of Ramp</th>
<th>Angle of Converg. at Ramp Curb</th>
<th>Angle of Converg. 2' off Pvm, Edge</th>
<th>Grades at Mane</th>
<th>Frwy, Direction</th>
<th>Ramp Type</th>
<th>Ramp Curvature</th>
<th>Curb Offset at Mane</th>
<th>Length of Ramp</th>
<th>Comments</th>
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<tbody>
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<td>Houston Area</td>
<td>Weslayan to SW Fry</td>
<td>610'-P</td>
<td>12°15'</td>
<td>12°15'</td>
<td>0%</td>
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<td>diamond</td>
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<td>2'</td>
<td>105'</td>
<td>4-lane, Frwy curve 10 right</td>
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<tr>
<td></td>
<td>Deer Sply to SW Fry</td>
<td>620'-P</td>
<td>12°</td>
<td>12°</td>
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<td>East</td>
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<td>tangent</td>
<td>2'</td>
<td>165'</td>
<td>Frwy, Grade past ramp at 6%</td>
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<td>Pulaski Rd Entrance to SW Expwy</td>
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<td>900'</td>
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<td>712'-T</td>
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<td>6°</td>
<td>1.0%</td>
<td>East</td>
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<td>3'</td>
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<td>Dempster to Eisenhower Expwy</td>
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<td>468'-P</td>
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<td>600'</td>
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<td>800'</td>
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<td>trumpet</td>
<td>directional</td>
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<td>450'</td>
<td>4-lane</td>
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<td>Wilshire to S. Bayshore</td>
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<td>5°</td>
<td>0%</td>
<td>South</td>
<td>from</td>
<td>Cloverleaf C-D road</td>
<td>Sea comment</td>
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<td>Broadway to S. Bayshore</td>
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<td>6°</td>
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<td>diamond</td>
<td>slight</td>
<td>2'</td>
<td>500'</td>
<td>Frwy curve slight right</td>
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67
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<th>Angle of Change at Exit Ramp</th>
<th>Grade at Ramp</th>
<th>Freeway Direction</th>
<th>Ramp Type</th>
<th>Curb Offset at Ramp</th>
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<td>6°15'</td>
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<td>slight right</td>
<td>diamond</td>
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<td>600'</td>
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<td>Jewel (69) to Grand Central</td>
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<td>5°</td>
<td>level</td>
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<td>1°</td>
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<td>right</td>
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<td>1200'</td>
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<td>Community Dr. to L. I. Exp.</td>
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<td>450'</td>
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<td>1°</td>
<td>-1.0%</td>
<td>East</td>
<td>connector</td>
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<td>left</td>
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<td>Brush Hollow Rd. to Northern St, Pky</td>
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<td>8°15'</td>
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<td>left</td>
<td>2'</td>
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<td>Brentwood to Daniel Boone Exp.</td>
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<td>+0.5%</td>
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<td>connector</td>
<td>slight right</td>
<td>left</td>
<td>2'</td>
<td>600'</td>
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</tbody>
</table>

Notes:
- Ramp grade is 4.1%.
- Ramp grade is 3.1%.
- Freeway grade is 8.5%.
- Ramp grade is 8.7%.
- Ramp grade is -7.6%.
- Ramp grade is -4.0%.
- Ramp grade is -5.8%.
- Ramp grade is -4.1%.
- Ramp grade is +3.0%.
- Ramp grade is +3.1%.
- Ramp grade is ±1.0%.

Comments:
- 2-lane freeway curve slight right.
## Appendix B

Sample Computer Output for Lag and Gap Analyses

<table>
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<tr>
<th>Ramp Veh. No.</th>
<th>Type</th>
<th>Arr Time</th>
<th>Type</th>
<th>Ramp Speed At Nose</th>
<th>Entry Lag Speed At Nose</th>
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<th>Gap 2</th>
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Appendix C

DEVELOPMENT OF PROBIT ANALYSIS
FOR INDIVIDUAL RECORDS

The development in this section follows closely a similar development in Finney. An adaptation for individual records is made of his development.

Consider the discrete probability density function \( Pr(r) = P^r Q^{1-r} \), \( r = 0, 1 \), defined for each number \( x \), where

\[ P = \Pr(t < x) = \int_{-\infty}^{x} f(\theta_1, \theta_2, t) \, dt \]

and \( f(\theta_1, \theta_2, x) \) is a probability density function. A sample of size \( k \) of \( x \)'s and \( r \)'s is obtained:

\( (x_1, r_1), (x_2, r_2), \ldots, (x_k, r_k) \). The likelihood of \( r_1, r_2, \ldots, r_k \) is proportional to \( e^L \), where

\[ L = \sum_{i=1}^{k} r_i \cdot \ln P_i + \sum_{i=1}^{k} (1-r_i) \cdot \ln Q_i. \]

The maximum likelihood estimates of \( \theta_1, \theta_2 \) are solutions of

\[ \frac{\partial L}{\partial \theta_1} = 0 \]

and

\[ \frac{\partial L}{\partial \theta_2} = 0. \]

Substituting for \( L \), these equations become

\[ \frac{\partial L}{\partial \theta_1} = \sum \frac{r}{P} \frac{\partial P}{\partial \theta_1} + \sum \frac{1-r}{Q} \frac{\partial Q}{\partial \theta_1} = \sum \frac{r-P}{PQ} \frac{\partial P}{\partial \theta_1} = 0 \]

and
(6) \( \frac{\partial L}{\partial \theta_2} = \sum \frac{r-P}{PQ} \frac{\partial P}{\partial \theta_2} = 0. \)

Let \( t_1, t_2 \) be first approximations to the M. L. E.'s of \( \theta_1, \theta_2 \). By the Taylor-Maclaurin expansion,

\[
(7) \frac{\partial L}{\partial \theta_1} \bigg|_{t_1, t_2} + \delta \theta_1 \frac{\partial^2 L}{\partial \theta_1 \partial \theta_1} \bigg|_{t_1, t_2} + \delta \theta_2 \frac{\partial^2 L}{\partial \theta_2 \partial \theta_2} \bigg|_{t_1, t_2} = 0; \\
(8) \frac{\partial L}{\partial \theta_2} \bigg|_{t_1, t_2} + \delta \theta_1 \frac{\partial^2 L}{\partial \theta_1 \partial \theta_2} \bigg|_{t_1, t_2} + \delta \theta_2 \frac{\partial^2 L}{\partial \theta_2 \partial \theta_2} \bigg|_{t_1, t_2} = 0. \\
\]

Similarly,

As in Finney (3), replace \( \frac{\partial^2 L}{\partial \theta_1 \partial \theta_1}, \frac{\partial^2 L}{\partial \theta_2 \partial \theta_2} \) and \( \frac{\partial^2 L}{\partial \theta_1 \partial \theta_2} \) by their expectations by putting \( r = P \) after differentiation. Substituting for \( L \), adjustments \( \delta \theta_1, \delta \theta_2 \) to \( t_1, t_2 \) are solutions of the equations

\[
(9) \delta \theta_1 \sum \frac{1}{PQ} \left( \frac{\partial P}{\partial \theta_1} \right)^2 \bigg|_{t_1, t_2} + \delta \theta_2 \sum \frac{1}{PQ} \left( \frac{\partial P}{\partial \theta_2} \right)^2 \bigg|_{t_1, t_2} = \sum \frac{r-P}{PQ} \frac{\partial P}{\partial \theta_1} \bigg|_{t_1, t_2} \\
(10) \delta \theta_1 \sum \frac{1}{PQ} \frac{\partial P}{\partial \theta_1} \bigg|_{t_1, t_2} + \delta \theta_2 \sum \frac{1}{PQ} \left( \frac{\partial P}{\partial \theta_2} \right)^2 \bigg|_{t_1, t_2} = \sum \frac{r-P}{PQ} \frac{\partial P}{\partial \theta_2} \bigg|_{t_1, t_2} \\
\]

A new pair of adjustments is found by recalculating (9) and (10) using \( t_1 + \delta \theta_1, \; t_2 + \delta \theta_2 \) in place of \( t_1, t_2 \). The iteration continues until the adjustments become as small as desired. The variances and
covariances of the estimates are obtained as the elements of the inverse of the matrix of coefficients of \( \delta \theta_1, \delta \theta_2 \) in (7) and (8), all terms being evaluated at the values \( t_1 + \delta \theta_1, t_2 + \delta \theta_2 \) obtained in the last iteration. No conditions for convergence are mentioned by Finney (4) and it appears difficult to obtain general conditions of this kind. Though modifications may be attempted along the lines described by Hartley (5), this problem is not investigated here.

In this thesis \( f \) is assumed to be such that

\[
P = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{(t-x)^2}{2\sigma^2}\right] dt
\]

Letting \( a = \frac{-\mu}{\sigma} \) and \( \beta = \frac{1}{\sigma} \) and \( Y = a + \beta x \), it is seen that

\[
P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y} \exp(-u^2) du
\]

\( Y \) is called the probit of \( P \).

Let \( z = \frac{\delta P}{\delta Y} = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}Y^2) \). Then

\[
\frac{\delta P}{\delta \alpha} = z
\]

and

\[
\frac{\delta P}{\delta \beta} = zx
\]

Let \( a, b \) be first approximations to the M. L. E.'s of \( \alpha, \beta \); then adjustments \( \delta a, \delta b \) to \( a, b \) are solutions of

\[
\delta a \sum \frac{z^2}{PQ} + \delta b \sum \frac{z^2 x}{PQ} x = \sum \frac{z^2}{PQ} \left( \frac{r-P}{z} \right),
\]

\[
\delta a \sum \frac{z^2 x}{PQ} + \delta b \sum \frac{z^2 x^2}{PQ} + \sum \frac{z^2}{PQ} \left( \frac{r-P}{PQ} \right) x.
\]

Let \( w = \frac{z^2}{PQ} \) so that the equations become

\[
72
\]
Let $\tilde{x} = \frac{\Sigma wx}{\Sigma w}$ and $a' = a + b\tilde{x}$. Then equation (17) becomes

$$\delta a\Sigma w + \delta b\Sigma wx = \Sigma w\left(\frac{r-P}{z}\right)$$

Let $\tilde{x} = \frac{\Sigma wx}{\Sigma w}$ and $a' = a + b\tilde{x}$. Then equation (17) becomes

$$\delta a\Sigma w + \delta b\Sigma wx = \Sigma w\left(\frac{r-P}{z}\right)$$

but $\delta a' = \delta a + \tilde{x}\delta b$, so the equation reduces to

$$\delta a'\Sigma w = \Sigma w\left(\frac{r-P}{z}\right)$$

Equation (18) becomes

$$\delta b\Sigma wx^2 = \Sigma w\left(\frac{r-P}{z}\right)x - \delta a\Sigma wx$$

$$= \Sigma w\left(\frac{r-P}{z}\right)x - (\delta a' - \tilde{x}\delta b)\Sigma wx$$

$$= \Sigma w\left(\frac{r-P}{z}\right)(x - \tilde{x}) + \delta b\left(\frac{\Sigma wx^2}{\Sigma w}\right)$$

or,

$$\delta b\Sigma w(x - \tilde{x})^2 = \Sigma w(x - \tilde{x})\left(\frac{r-P}{z}\right).$$

Let $y = Y + \frac{r-P}{z}$; $y$ is called the working probit. Then (20) becomes

$$\delta a'\Sigma w = \Sigma w(y - Y)$$

$$= \Sigma wy - a'\Sigma w$$

from which

$$a' + \delta a' = \frac{\Sigma wy}{\Sigma w} = \tilde{y}.$$

Proceeding similarly with (22),

$$\delta b\Sigma w(x - \tilde{x})^2 = \Sigma w(x - \tilde{x})[y - a' - b(x - \tilde{x})]$$

so that
(26) \( (b + \delta b) \sum w(x-x)^2 = \sum w(x-x) (y-y) \)

and hence

(27) \( b + \delta b = \frac{\sum w(x-x) (y-y)}{\sum w(x-x)^2} \).

Thus improved estimates of \( \alpha, \beta \) are obtained as the coefficients of a weighted linear regression of \( y \) on \( x \). With the improved estimates another iteration is carried out, and so on until the desired precision is obtained.

Throughout this thesis, \( r \) is defined by \( r = 1 \) if a gap (or lag) is accepted, \( r = 0 \) if a gap (or lag) is rejected. The \( x \)-variable is taken as log (lag) or log (gap) or, in the analyses of Table 4, as a linear combination of log (lag) and ramp velocity.

There are a number of \( \chi^2 \) tests monitoring certain features of the fitted model.

In cases where one probit line \( Y = \alpha + \beta x \) is estimated, a \( \chi^2 \) statistic with \( k-2 \) degrees of freedom provides a test of linearity:

(28) \( \chi^2 = \sum w(y-y)^2 - \left[ \frac{\sum w(x-x) (y-y)}{\sum w(x-x)^2} \right]^2 \).

If two probit lines are estimated and constrained to be parallel, a \( \chi^2 \) with \( k_1 + k_2 - 3 \) degrees of freedom for linearity and a \( \chi^2 \) with 1 degree of freedom for parallelism may be computed; if \( j \) lines are estimated and constrained to be parallel, the linearity \( \chi^2 \) has \( \sum_{i} k_i - j - 1 \) degrees of freedom and the parallelism \( \chi^2 \) has \( j-1 \) degrees of freedom.

If two parallel probit lines are estimated as \( Y_1 = \alpha_1 + \beta x_1 \) and \( Y_2 = \alpha_2 + \beta x_2 \) and \( x_1 \) and \( x_2 \) are measured on the log(stimulus) scale, the relative potency \( R_{1,2} \) of stimulus one to stimulus two is estimated as the antilogarithm of the horizontal distance between the two lines, or as \( 10^m \), where

(29) \( m = \bar{x}_2 - \bar{x}_1 - \frac{\bar{y}_2 - \bar{y}_1}{b} \).

As a result of Fieller's Theorem, confidence limits on \( R_{1,2} \) may be assessed as \( 10^{m_1}, 10^{m_u} \), where
(30) \( m_1, m_u = \bar{x}_2 - \bar{x}_1 + \frac{1}{1-g} \left\{ (m-\bar{x}_2 + \bar{x}_1) + \frac{t}{b} \left( (1-g) \sum \left( \frac{1}{S_w} \right) + (m-\bar{x}_0 + \bar{x}_1) \right) \right\}^{1/2} \),

where \( g = \frac{t^2}{b^2 \sum Sx^2} \) and \( t \) is the normal deviate for the desired significance level. The summations are over sets one and two. The notation

\[
Sw = \sum_{i=1}^{k} w_i, \quad Sx^2 = \sum_{i=1}^{k} w_i (x_i - \bar{x})^2, \quad Sxy = \sum_{i=1}^{k} w_i (x_i - \bar{x})(y_i - \bar{y}), \quad \text{etc., are used in this thesis where convenient and unambiguous.}
\]

When one probit line \( Y = a + bx \) is estimated, the estimate of \( \mu \) is \( m = -\frac{\bar{Y}}{b} \); the 50% point of the tolerance distribution is then estimated as \( m = 10^m \). Confidence limits on \( m \) are estimates as \( 10^{m_1}, 10^{m_u} \), where \( m_1, m_u \) are the roots of the quadratic

(31) \( Y^2 = t^2 V(Y) = t^2 \left[ \frac{1}{Sw} + \frac{(x-x)^2}{Sx^2} \right] \) in \( x \). Similarly, if \( Y = a + b_1 x_1 + b_2 x_2 \) is estimated, the 50% point for \( x_1 \) at a given \( x_2 \) is obtained as

(32) \( m = -\frac{-a - b_2 x_2}{b_1} \)

and confidence limits on \( m \) are obtained as the roots of the quadratic

(33) \( Y^2 = t^2 \left[ \frac{1}{Sw} + (x_1-\bar{x}_1)^2 V_{11} + (x_2-\bar{x}_2)^2 V_{22} + 2(x_1-\bar{x}_1)(x_2-\bar{x}_2)V_{12} \right] \),

where \( V_{ij} \) is the \( i, j \)-th element of

\[
V = \begin{pmatrix}
Sx_1^2 & Sx_1 x_2 \\
Sx_1 x_2 & Sx_2^2
\end{pmatrix}
\]
APPENDIX D

Lag and Gap Acceptance for U.S. Ramps

Figs. 19-23

(See also Figure 7)
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 19
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 20
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 21
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 22
LAG AND GAP ACCEPTANCE REGRESSIONS FOR U.S. RAMPS

FIGURE 23
APPENDIX E

Gap Acceptance with Confidence Limits

Figs. 24-28

(See also Figure 8)
GAP ACCEPTANCE WITH CONFIDENCE LIMITS

FIGURE 24
GAP ACCEPTANCE WITH CONFIDENCE LIMITS

FIGURE 25
GAP ACCEPTANCE WITH CONFIDENCE LIMITS

FIGURE 26
GAP ACCEPTANCE WITH CONFIDENCE LIMITS

FIGURE 28
APPENDIX F

Gap Acceptance for Multiple Vehicle Merges

Figs. 29-33

(See also Figure 9)
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 29
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 30
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 31
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 32
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES

FIGURE 33
APPENDIX G

Gap Acceptance for Multi. Veh. Merges (Constrained)

Figs. 34-38

(See also Figure 10)
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES (CONSTRAINED)

FIGURE 34
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES (CONSTRAINED)

FIGURE 35
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES (CONSTRAINED OR PARALLEL)

FIGURE 56
GAP ACCEPTANCE FOR MULTIPLE VEHICLE Merges (Constrained)

Figure 37
GAP ACCEPTANCE FOR MULTIPLE VEHICLE MERGES (CONSTRAINED OR PARALLEL)

FIGURE 38
APPENDIX H

Gap Acceptance for Fast and Slow Merging Vehicles

Figs. 39-43

(See also Figure 13)
GAP ACCEPTANCE FOR FAST & SLOW MERGING VEHICLES

FIGURE 39
GAP ACCEPTANCE FOR FAST & SLOW MERGING VEHICLES

FIGURE 40
GAP ACCEPTANCE FOR FAST & SLOW MERGING VEHICLE

FIGURE 42
GAP ACCEPTANCE FOR FAST & SLOW MERGING VEHICLES

FIGURE 43
APPENDIX I

Effect of Speed of Ramp Vehicle on Lag Acceptance

Figs. 44-48

(See also Figure 17)
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 44
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 45
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 46
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 47
EFFECT OF SPEED OF RAMP VEHICLE ON LAG ACCEPTANCE

FIGURE 48
APPENDIX J

Effect of Relative Speed on Lag Acceptance

Figs. 49-53

(See also Figure 18)
EFFECT OF RELATIVE SPEED ON LAG ACCEPTANCE

FIGURE 49
EFFECT OF RELATIVE SPEED ON LAG ACCEPTANCE

FIGURE 50
EFFECT OF RELATIVE SPEED ON LAG ACCEPTANCE

FIGURE 51
EFFECT OF RELATIVE SPEED ON LAG ACCEPTANCE

FIGURE 52
EFFECT OF RELATIVE VELOCITY ON LAG ACCEPTANCE

FIGURE 53
### Two Variable Probit Analysis

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<td>CHI-10</td>
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| DET-01 | -5.2606 | 3.1123    | 0.1645 | 1.0642  | 2.5874    | -0.0659 |
| DET-02 | -1.2709 | 1.5500    | 0.0606 | 0.7891  | 1.6788    | -0.0387 |
| DET-04 | 0.8886  | 1.6084    | 0.0074 | 0.6698  | 1.6900    | -0.0333 |
| DET-05 | -4.5808 | 2.2830    | 0.1210 | 1.8385  | 3.7743    | -1.1606 |
| DET-06 | -1.0907 | 2.2521    | 0.0659 | 1.6773  | 2.3232    | -0.0611 |
| DET-07 | -1.705  | 1.0526    | 0.0422 | 0.3989  | 1.0485    | -0.0412 |
| DET-08 | -0.0526 | 0.7780    | 0.0502 | 0.3785  | 0.7481    | 0.0454  |

| STL-01 | -2.0329 | 1.5039    | 0.1094 | 0.0039  | 1.4178    | -0.0566 |
| STL-02 | 0.2657  | 1.3859    | 0.0021 | 0.1304  | 1.4753    | -0.0292 |
| STL-03 | -1.0301 | 1.8347    | 0.0746 | 0.5797  | 1.7079    | -0.0443 |