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16. Abstract

The knowledge of speed and headway distributions is essential in microscopic traffic flow studies because speed and headway are both fundamental microscopic characteristics of traffic flow. For microscopic simulation models, one key process is the generation of entry vehicle speeds and vehicle arrival times. It is helpful to find desirable mathematical distributions to model individual speed and headway values, because the individual vehicle speed and arrival time in microscopic simulations are usually generated based on some form of mathematical models. Traditionally, distributions for speed and headway are investigated separately and independent of each other. However, this traditional approach ignores the possible dependence between speed and headway. To address this issue, the research presents a methodology to construct bivariate distributions to describe the characteristics of speed and headway. Based on the investigation of freeway speed and headway data measured from the loop detector data on IH-35 in Austin, it is shown that there exists a weak dependence between speed and headway.

The research first proposes skew-t mixture models to capture the heterogeneity in speed distribution. Finite mixture of skew-t distributions can significantly improve the goodness of fit of speed data. To develop a bivariate distribution to capture the dependence and describe the characteristics of speed and headway, this study proposes a Farlie-Gumbel-Morgenstern (FGM) approach to construct a bivariate distribution to simultaneously describe the characteristics of speed and headway. The bivariate model can provide a satisfactory fit to the multimodal speed and headway distribution. Overall, the proposed methodologies in this research can be used to generate more accurate vehicle speeds and vehicle arrival times by considering their dependence on each other when developing microscopic traffic simulation models.

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A MULTIVARIATE ANALYSIS OF FREEWAY SPEED AND HEADWAY DATA

By

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December, 2013

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ABSTRACT

The knowledge of speed and headway distributions is essential in microscopic traffic flow studies because speed and headway are both fundamental microscopic characteristics of traffic flow. For microscopic simulation models, one key process is the generation of entry vehicle speeds and vehicle arrival times. It is helpful to find desirable mathematical distributions to model individual speed and headway values, because the individual vehicle speed and arrival time in microscopic simulations are usually generated based on some form of mathematical models. Traditionally, distributions for speed and headway are investigated separately and independent of each other. However, this traditional approach ignores the possible dependence between speed and headway.

To address this issue, the research presents a methodology to construct bivariate distributions to describe the characteristics of speed and headway. Based on the investigation of freeway speed and headway data measured from the loop detector data on IH-35 in Austin, it is shown that there exists a weak dependence between speed and headway.

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Farlie-Gumbel-Morgenstern (FGM) approach to construct a bivariate distribution to simultaneously describe the characteristics of speed and headway. The bivariate model can provide a satisfactory fit to the multimodal speed and headway distribution. Overall, the proposed methodologies in this research can be used to generate more accurate vehicle speeds and vehicle arrival times by considering their dependence on each other when developing microscopic traffic simulation models.

TABLE OF CONTENTS

Page

LIST OF FIGURES

Page

LIST OF TABLES

Page

CHAPTER I

INTRODUCTION

Speed is a fundamental measure of traffic performance of a highway system (May, 1990). Most analytical and simulation models of traffic either produce speed as an output or use speed as an input for travel time, delay, and level of service determination (Park et al., 2010). It is desirable to find an appropriate mathematical distribution to describe the measured speeds, because in some microscopic simulations the individual vehicle speed needs to be determined according to some form of mathematical model during vehicle generation (Park et al., 2010).

Headway is an important flow characteristic and headway distribution has applications in capacity estimation, driver behavior studies and safety analysis (May, 1990). The distribution of headway determines the requirement and the opportunity for passing, merging, and crossing (May, 1990). The headway distribution under capacity-flow conditions is also a primarily factor in determining the capacity of systems. Moreover, a key component in many microscopic simulation models is to generate entry vehicle headway in the simulation process. To generate accurate vehicle arrival times to the simulated network, it is necessary to use appropriate mathematical distributions to model headway.

As described above, the knowledge of speed and headway is necessary because these variables are fundamental measures of traffic performance of a highway system. Therefore, developing reliable and innovative analytical techniques for analyzing these variables is very important. The primary goal of this research is to develop some new methodologies for the analysis of microscopic freeway speed and headway data.

1.1 Statement of the Problem

This research consists of two parts. The first part concerns the heterogeneity problem in freeway vehicle speed data. If the characteristics of speed data are homogeneous, speed can be generally modeled by normal, log-normal and gamma distributions. However, if the speed data exhibit excess skewness and bimodality (or heterogeneity), unimodal distribution function does not give a satisfactory fit. Thus, the mixture model (composite model) has been considered by May (1990) for traffic stream that consists of two classes of vehicles or drivers. So far, the mixture models used in previous studies to fit bimodal distribution of speed data considered normal density as the specified component; therefore, it is useful to investigate other types of component density for the finite mixture model.

The second parts concern the dependence between freeway speed and headway data. Traditionally, the dependence between speed and headway is ignored in the microscopic simulation models. As a result, the same headway distribution may be assumed for different speed levels and this assumption neglects the possible variability of headway distribution across speed values. Moreover, a number of developed microscopic simulation models generate vehicle speeds and vehicle arrival times as independent inputs to the simulation process. Up to date, only a few studies have been directed at exploring the dependence between speed and headway. Considering the potential dependence between speed and headway, it is useful to construct bivariate distribution models to describe the characteristics of speed and headway. Compared with one dimensional statistical models representing speed or headway separately, bivariate distributions have the advantage that the possible correlation between speed and headway is taken into consideration. Given this advantage, it is necessary to construct bivariate distributions to improve the accuracy or validity of microscopic simulation models.

1.2 Research Objectives

The primary goal of this research is to develop new methodologies for analyzing the characteristics of speed and headway. To accomplish this goal, following objectives are planned to be addressed in this research.

1. To address the heterogeneity problem in freeway vehicle speed data, we apply skew-normal and skew-t mixture models to capture excess skewness, kurtosis and bimodality present in speed distribution. Skew-normal and skew-t distributions are known for their flexibility, allowing for heavy tails, high degree of kurtosis and asymmetry. To investigate the applicability of mixture models with skew-normal and

skew-t component density, we fit a 24-hour speed data collected on IH-35 using skew-normal and skew-t mixture models with the Expectation Maximization type algorithm.

2. To construct bivariate distribution of speed and headway, we examine the dependence structure between the two variables. Three correlation coefficients (i.e., Pearson correlation coefficient, Spearman's rho and Kendall's tau) are used to evaluate the dependence between speed and headway.

3. To develop a bivariate distribution for capturing the dependence and describing the characteristics of speed and headway simultaneously, the Farlie-Gumbel-Morgenstern (FGM) approach is proposed.

1.3 Outline of the Research

The rest of this research is organized as follows:

Chapter II overviews various mathematical models that have been used for describing speed and headway distributions. Some studies that focused on the dependence between speed and headway are also discussed.

Chapter III provides the characteristics of the traffic dataset used throughout in the research. A preliminary analysis is conducted to investigate the dependence structure between speed and headway.

Chapter IV applies skew-t mixture models to fit freeway speed data. This chapter shows that finite mixture of skew-t distributions can significantly improve the goodness of fit of speed data and better account for heterogeneity in the data.

Chapter V explores the applicability of the FGM approach to address the heterogeneity problem in speed and headway data. This chapter shows that the bivariate model can provide a satisfactory fit to the speed and headway data.

Chapter VI summarizes the major results of in this research. General conclusions and recommendations for future research are presented.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

This chapter first provides a review of mathematical models for speed and headway. Specifically, different speed and headway distributions proposed in the past studies are introduced. Then, we discuss some research focused on the dependence between speed and headway.

2.2 Speed Distributions

Previously, normal, log-normal and other forms of distribution have been used to fit freeway speed data. Leong (1968) and McLean (1979) proposed that speed data approximately follow a normal distribution when flow rate is light. Haight and Mosher (1962) showed that the log-normal distribution is proper for speed data. Gerlough and Huber (1976) and Haight (1965) have used normal, log-normal and gamma distributions to model vehicular speed. Compared with normal distribution, log-normal and gamma distributions have the capacity to accommodate the right skewness and eliminate negative speed values generated by normal distribution. If the speed data exhibit excess skewness and bimodality, unimodal distribution function does not give a satisfactory fit; thus, several researchers used the mixture model to fit the distribution of speed. When the traffic stream consists of two vehicle types, the composite distribution has been proposed by May (1990). He also suggested that the vehicle speeds for subpopulations follow normal or lognormal distributions. Dey et al. (2006) introduced a new parameter, spread ratio to predict the shape of the speed curve. He stated that the bimodal speed distribution curve consists of a mixture of two-speed fractions, lower fraction and upper fraction. Ko and Guensler (2005) did a similar study by characterizing the speed data with two different normal components, one for congested and the other for non-congested speeds. The congestion characteristics can be identified based on the speed distribution. Recently, Park et al. (2010) explored the distribution of 24-hour speed data with a g-component normal mixture model. Jun (2010) investigated traffic congestion trends by speed patterns during holiday travel periods using the normal mixture model.

2.3 Headway Distributions

Many headway models have been proposed and these models can be classified into two types: single distribution models and mixed models. For single distribution models, exponential (Cowan, 1975), normal, gamma, lognormal and log-logistic distributions (Yin et al., 2009) have been studied to model headway. The representatives of mixed models are Cowan M3 model (Luttinen, 1999), M4 model (Hoogendoorn and Bovy, 1998), the generalized queuing model and the semi-Poisson model (Wasielewski, 1979). Zhang et al. (2007) performed a comprehensive study of the performance of typical headway models using the headway data recorded from general-purpose lanes.

2.4 Dependence between Speed and Headway

There have been some studies that focused on the dependence between speed and headway. Luttinen (1992) found out that speed limit and road category have a considerable effect on the statistical properties of vehicle headways. WINSUM and Heino (1996) investigated the time headway and braking response during car-following. Taieb-Maimon and Shinar (2001) conducted a study to investigate drivers' following headways in car-following situation and the results showed that drivers adjusted the distance headways in relation to speed. Dey and Chandra (2009) proposed two statistical distributions for modeling the gap and headway in the steady car-following state. Brackstone et al. (2009) found that there is a limited dependence of following headway on speed and the most successful relationship fit of headway and speed is an inverse relationship. Yin et al. (2009) also studied the dependence of headway distributions on the traffic condition (speed pattern) and concluded that different headway models should be used for distinct traffic conditions (speed patterns).

2.5 Summary

From the above discussion, there are several current issues existing in modeling the speed and headway data. First, when modeling multimodal distribution of speed data, the mixture models used in previous studies extensively considered normal density as the specified component; therefore, other types of component density were not fully investigated. Second, considering the possible dependence between speed and headway, there were very few studies focusing on constructing bivariate distribution models to describe speed and headway simultaneously.

CHAPTER III

DATA INTRODUCTION AND PRELIMINARY ANALYSIS

3.1 Introduction

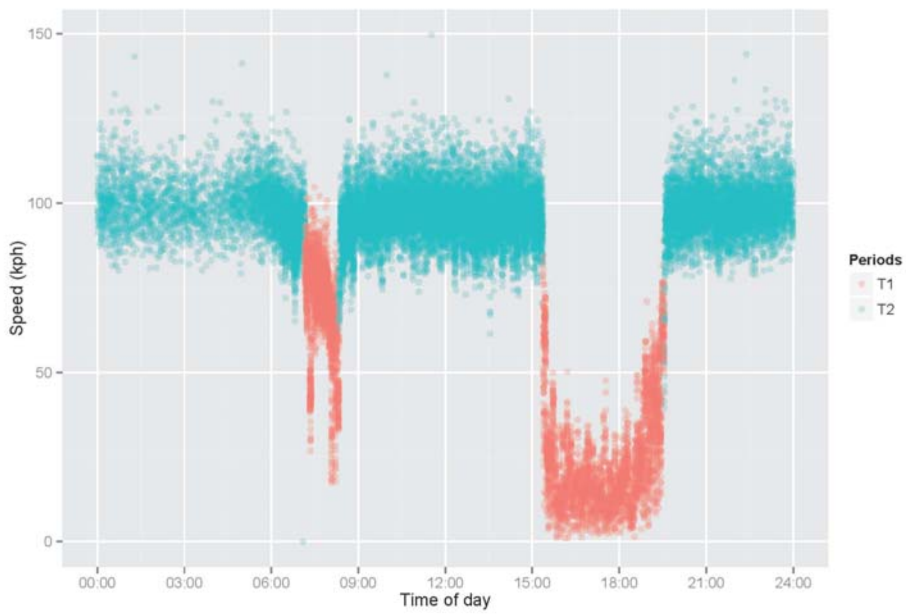
As discussed in Chapter I, the main objective of this research is to develop new methodologies for analyzing the characteristics of freeway speed and headway data. The traffic data analyzed in this research are the microscopic traffic variables (i.e., individual speed and headway observations) measured from the loop detector data. The study site is on IH-35 in Austin, Texas. This chapter introduces the characteristics of the traffic dataset which is used throughout in the research. A preliminary analysis is conducted to investigate the dependence structure between observed speed and headway data.

3.2 Data Description

The dataset was collected at a location on IH-35. IH-35 has four lanes in the southbound direction and the free flow speed is 60 mile/hour (or 96.56 kilometer/hour) for all types of vehicles. Due to the heavy traffic demand and a large volume of heavy vehicles, the data collection site is typically congested during the morning and afternoon peak hours. The detector records vehicle arrival time, presence time, speed, length, and classification for each individual vehicle (Ye et al., 2006). This dataset was analyzed in some previous studies (Ye and Zhang, 2009). The data have 27920 vehicles with recorded speed values, arrival times and vehicle lengths in a 24-hour period (from 00:00 to 24:00, December 11, 2004), including 24011 (86%) passenger vehicles and 3909 (14%) heavy vehicles. For this dataset, the headway value between two consecutive vehicles is the elapsed time between the arrivals of a pair of vehicles. The arrival times were recorded in second (s); the observed speeds were recorded in meter/second; and the vehicle lengths were recorded in meter (m). To compare the result of this work with some previous studies, we convert the meter/second to kilometer/hour (kph). We also assume that 24-hour period (T) consists of two time periods: the peak time period (T1) which contains two sub-periods 07:10-08:20 and 15:22-19:33; while the off peak period (T2) includes two sub-periods 08:20-15:22 and 19:33-07:10.

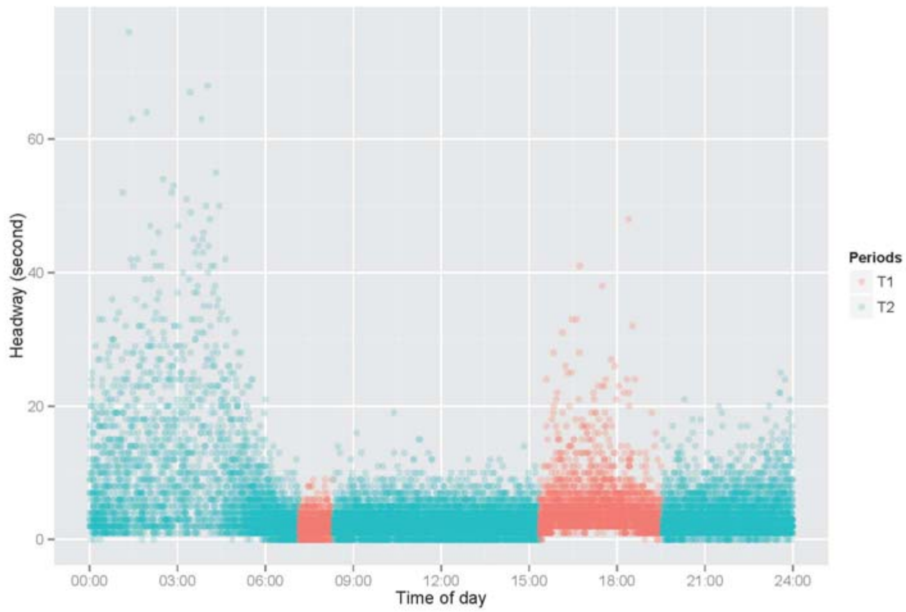
3.3 Preliminary Analysis

Figure 3.1 (a), (b) and (c) display the scatter plots of speed, headway and vehicle length by time of day for each time period. Because of large samples in the dataset, semi-transparent points are used to alleviate some of the over-plotting in Figure 3.1. Figure 3.1 (c) indicates that the observed vehicles seem to consist of two sub-populations: one at about 5 meters, representing passenger vehicles, and the other at about 22 meters, representing trucks and buses. Previously, Zhang et al. (2008) estimated large truck volume using loop detector data collected from IH-35, and they classified vehicles into two categories: short vehicles (smaller than 12.2 m (40 feet)) and long vehicles (larger than or equal to 12.2 m (40 feet)). In order to see the changing pattern of vehicle composition over the time, we calculate the hourly percentage of long vehicles (greater than or equal to 12.2 m), which is shown in Figure 3.1 (d). It can be observed that the proportion of long vehicles is relatively high between 00:00 and 6:00 compared with other time periods of the day.

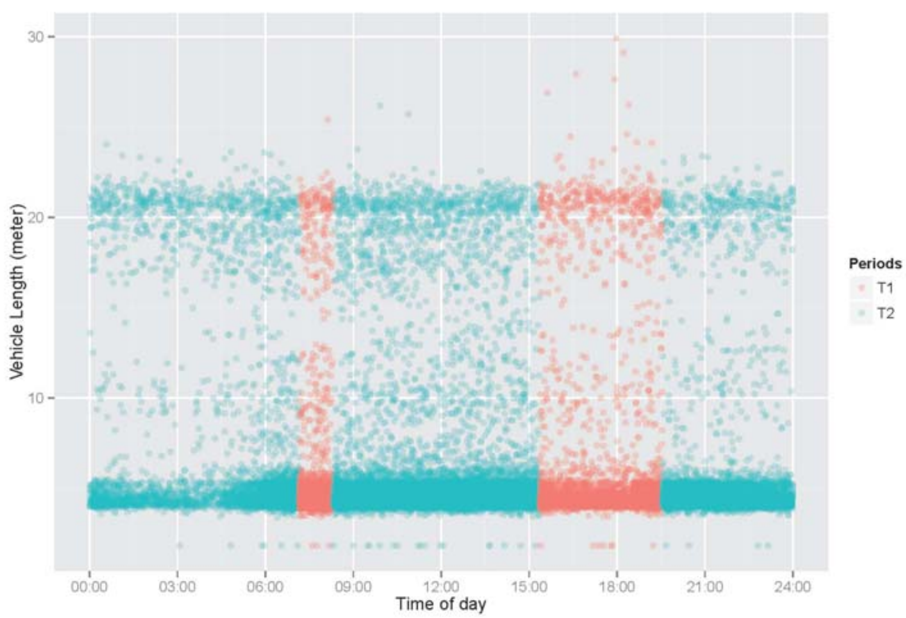


(a)

Figure 3.1 (a) speed scatter plots by time of day; (b) headway scatter plots by time of day; (c) vehicle length scatter plots by time of day; (d) hourly percentage of long vehicles by time of day.

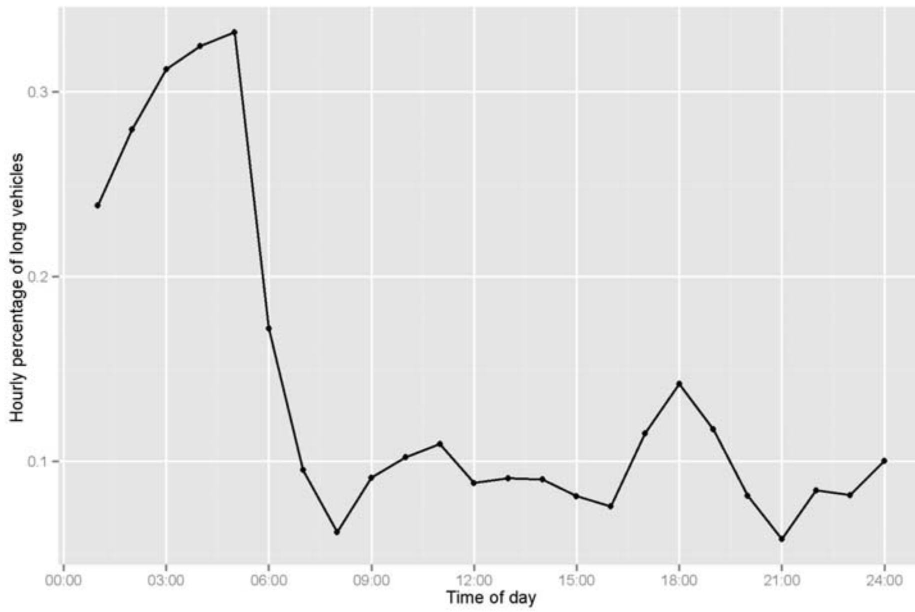


(b)



(c)

Figure 3.1 Continued



(d)

Figure 3.1 Continued

From Figure 3.1 (a), we can see that the speed data exhibit heterogeneity and the main cause for this heterogeneity is different traffic flow conditions over the 24-hour period. Since the characteristics of speed data are heterogeneous, the mixture models are used to capture bimodality present in speed distribution. Then, we examine the correlation between speed and headway. Since the 24-hour traffic data in the study consist of distinct traffic flow conditions, it is useful to evaluate the dependence between vehicle speed and headway under different traffic conditions. As discussed above, we divided the 24-hour traffic data into two time periods (i.e., the peak period T1 and the off-peak period T2) based on corresponding traffic conditions. For each time period, three correlation coefficients are used to evaluate the dependence. These three measures of dependence are Pearson correlation coefficient (PCC), Spearman's tau (SCC), and Kendall's pho (KCC). The summary statistics of speed and headway for different time periods are given in Table 3.1.

Table 3.1 Summary statistics of speed and headway for different time periods

	T (24 hours)		T1 (07:10-08:20 and 15:22-19:33)		T2 (08:20-15:22 and 19:33-07:10)	
	Speed	Headway	Speed	Headway	Speed	Headway
Min.	0	0 [a]	1.01	0	0	0
1st Quantile	84.74	1	18.22	2	92.38	1
Median	94.57	2	37.76	2	97.09	2
Mean	85.3	3.1	42.71	3.15	97.24	3.08
3rd Quantile	100.4	3	68.57	4	101.95	3
Max.	149.69	76	104.72	48	149.69	76
Number of vehicles	27919		6114		21805	
PCC	-0.054		-0.469		0.116	
KCC	0.003		-0.488		0.135	
SCC	0.011		-0.635		0.186	

Note: [a] Headway values are less than 0.5s.

PCC measures the linear relationship between two continuous variables. It is defined as the ratio of the covariance of the two variables to the product of their respective standard deviations:

$$\text{PCC} = \frac{Cov(x, y)}{\sigma_x \sigma_y} \tag{3.1}$$

where σ_x and σ_y are the standard deviations of variables x and y.

SCC is a rank-based version of the PCC and it can be computed as:

$$\text{SCC} = \frac{\sum_{i=1}^{n}(rank(x_i) - \overline{rank(x)})(rank(y_i) - \overline{rank(y)})}{\sqrt{\sum_{i=1}^{n}(rank(x_i) - \overline{rank(x)})^2 \sum_{i=1}^{n}(rank(y_i) - \overline{rank(y)})^2}} \tag{3.2}$$

where $rank(x_i)$ and $rank(y_i)$ are the ranks of the observation x_i and y_i in the sample.

Similar to SCC, KCC is designed to capture the association between two measured quantities. KCC quantifies the discrepancy between the number of concordant and discordant pairs. Its estimate can be expressed as follows:

$$\text{KCC}=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\text{sgn}(x_i - x_j)\,\text{sgn}(y_i - y_j)}{\frac{1}{2}n(n-1)} \tag{3.3}$$

where $\text{sgn}(x_i - x_j) = \begin{cases} 1 & \text{if } (x_i - x_j) > 0 \\ 0 & \text{if } (x_i - x_j) = 0 \\ -1 & \text{if } (x_i - x_j) < 0 \end{cases}$ and $\text{sgn}(y_i - y_j) = \begin{cases} 1 & \text{if } (y_i - y_j) > 0 \\ 0 & \text{if } (y_i - y_j) = 0 \\ -1 & \text{if } (y_i - y_j) < 0 \end{cases}$.

Note that the PCC, KCC, and SCC are -0.469, -0.488 and -0.635 between speed and headway for peak period T1, suggesting a moderate inverse relationship between these two traffic variables. Since speed and headway values in peak period T1 were observed under congested traffic conditions, it is reasonable to consider most of the headway values in time period T1 as following headways. From Figure 3.2, it is observed that headway increases as speed decreases, and the relationship can be split into two regimes. The time headway is approximately stable when speed is above 20 kph in the first regime. In the second regime when speed is below 20 kph, the time headway increases significantly as speed decreases. The findings from Figure 3.2 are consistent with the results reported in a study conducted by Brackstone et al. (2009). In their study, it is shown that there is a limited dependence of following headway on speed: the most successful relationship fit of headway and speed is an inverse relationship. Interestingly, KCC is 0.135 between speed and headway for off-peak period T2, indicating a positive dependence. This is reasonable because as headway values become larger during the off peak period, fewer vehicles are on the road and it is expected to see that vehicle speeds increase accordingly.

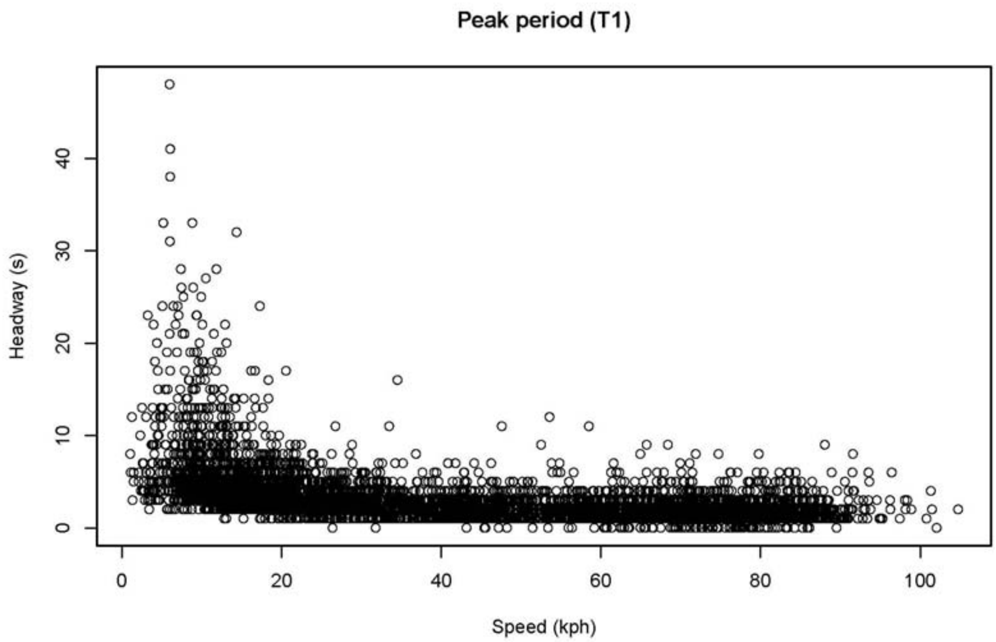


Figure 3.2 Scatter plot of speed and headway for peak period (T1).

3.4 Summary

This chapter described the characteristics of traffic data collected on IH-35. As shown in Figure 3.1 (a), the speed data are heterogeneous and to capture the bimodality present in the speed distribution, Chapter IV proposes skew-t mixture models to fit freeway speed data. Besides, the data analysis indicates that the two microscopic traffic variables (speed and headway) are correlated under different traffic conditions and to construct bivariate distribution of speed and headway, the FGM approach is proposed in the Chapter V.

CHAPTER IV

METHODOLOGY I: MIXTURE MODELING OF FREEWAY SPEED DATA[1]

4.1 Introduction

An appropriate mathematical distribution can help describing speed characteristics and is also useful for developing and validating microscopic traffic simulation models. To accommodate the heterogeneity in speed data, the mixture models used in previous studies extensively considered normal density as the specified component; therefore, other types of component density were not fully investigated. To capture excess skewness, kurtosis and bimodality present in speed distribution, we propose skew-normal and skew-t mixture models to fit freeway speed data. This chapter shows that finite mixture of skew-t distributions can significantly improve the goodness of fit of speed data and better account for heterogeneity in the data.

4.2 Finite Mixture Models

In this chapter, it is assumed that the speed data are independent and identically distributed (i.i.d.) realizations from a random variable which follows either a mixture of g-component normal, skew-normal or skew-t mixture model. The mixture model is

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[1] Reprinted with permission from "Use of skew-normal and skew-t distributions for mixture modeling of freeway speed data" by ZOU, Y., & ZHANG, Y., 2011. Transportation Research Record, 2260, 67-75, Copyright [2011] by the Transportation Research Board.

widely used in modeling bimodal speed distribution to account for the heterogeneity. The normal, skew-normal and skew-t mixture models are briefly introduced in this section:

The normal mixture model for the vehicle speed has the following probability density function:

$$f(x \mid w_k, \xi_k, \sigma_k^2) = \sum_{k=1}^{N} w_k NL(x \mid \xi_k, \sigma_k^2) \tag{4.1}$$

$$NL(x \mid \xi_k, \sigma_k^2) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(x-\xi_k)^2}{2\sigma_k^2}\right) \tag{4.2}$$

The expectation and variance of a normal distribution can be written as:

$$E(x) = \xi_k \tag{4.3}$$

$$Var(x) = \sigma_k^2 \tag{4.4}$$

where N is the number of components, w_k is the weight of component k, with $1 > w_k > 0$ and $\sum_{k=1}^{N} w_k = 1$, ξ_k is the location parameter, σ_k^2 is the scale parameter, and $NL(x \mid \xi_k, \sigma_k^2)$ is the normal density function with mean ξ_k and variance σ_k^2.

The skew-normal distribution was first developed by Azzalini (1985). The probability density function for the skew-normal mixture model is given by:

$$f(x \mid w_k, \xi_k, \sigma_k^2, \lambda_k) = \sum_{k=1}^{N} w_k SN(x \mid \xi_k, \sigma_k^2, \lambda_k) \tag{4.5}$$

$$SN(x \mid \xi_k, \sigma_k^2, \lambda_k) = \frac{2}{\sigma_k} \phi\left(\frac{x-\xi_k}{\sigma_k}\right) \Phi\left(\lambda_k \frac{x-\xi_k}{\sigma_k}\right) \tag{4.6}$$

The expectation and variance of a skew-normal distribution are given by

$$E(x) = \xi_k + \sigma\delta_k \sqrt{\frac{2}{\pi}} \tag{4.7}$$

$$Var(x) = \sigma_k^2 \left(1 - \frac{2\delta_k^2}{\pi}\right) \tag{4.8}$$

where $\delta_k = \frac{\lambda_k}{\sqrt{1+\lambda_k^2}}$, λ_k is the skewness parameter, $\phi(\cdot)$ and $\Phi(\cdot)$ are, the standard normal density and cumulative distribution function, and $SN(x \mid \xi_k, \sigma_k^2, \lambda_k)$ is the skew-normal density function. The mean and variance of $SN(x \mid \xi_k, \sigma_k^2, \lambda_k)$ are given in equations (4.7) and (4.8), respectively.

It can be shown that the excess kurtosis of a skew-normal distribution is limited to the interval [0, 0.8692]. Later, the skew-t distribution was introduced by Azzalini and Capitanio (2003) to allow for a higher degree of kurtosis. The skew-t mixture model can be written as follows:

$$f(y \mid w_k, \xi_k, \sigma_k^2, \lambda_k, v) = \sum_{k=1}^{N} w_k ST(y \mid \xi_k, \sigma_k^2, \lambda_k, v) \tag{4.9}$$

$$ST(y \mid \xi_k, \sigma_k^2, \lambda_k, \nu) = \frac{2}{\sigma_k} t_\nu(x_y) T_{\nu+1}\left(\lambda_k x_y \sqrt{\frac{\nu+1}{\nu+x_y^2}}\right) \tag{4.10}$$

where ν is the degrees of freedom, $x_y = (y-\xi_k)/\sigma_k$, t_ν and T_ν represent the standard Student-t density and cumulative function with ν degrees of freedom, and $ST(y \mid \xi_k, \sigma_k^2, \lambda_k, \nu)$ is the skew-t density function. Also, it can be shown that the skew-t distribution converges to a skew-normal distribution when $\nu \to \infty$ (ν tends to infinity).

4.3 Model Estimation Method

There are various methods available for estimating a mixture model. The method of moments was first used by Pearson in the early days of mixture modeling. The maximum likelihood estimation with Expectation Maximization (EM) algorithm and Bayesian estimation become the most widely applied methods when large calculations can be easily done by powerful computers. Assuming the number of components is known, Bayesian approach can be implemented with data augmentation and Markov Chain Monte Carlo (MCMC) estimation procedure using Gibbs sampling techniques (Zou et al, 2012). However, one of the main drawbacks of MCMC procedures is that they are generally computationally demanding, and it can be difficult to diagnose convergence (Zou et al, 2012). Furthermore, the label switching is another difficulty and has to be addressed explicitly when using a Bayesian approach to conduct parameter estimation and clustering (Frühwirth-Schnatter, 2006).

Since the label switching is of no concern for maximum likelihood estimation, the maximum likelihood method is adopted for estimation of finite mixture of skew-normal and skew-t distributions in this study. The EM algorithm was introduced by Dempster et al. (1977) and there are two extensions of it: the Expectation/Conditional Maximization Either (ECME) and the Expectation/Conditional Maximization (ECM) algorithms. Among the three algorithms, the ECM algorithm converges more slowly than the EM algorithm, but consumes less processing time in computer. The ECME algorithm has the greatest speed of convergence as well as the least processing time; moreover, it preserves the stability with monotone convergence. Thus, the ECME algorithm is chosen for the estimation of the parameters here.

4.4 Modeling Results

We apply normal, skew-normal and skew-t mixture models with an increasing number of components (g =2,…,6) to the 24-hour speed data described in Chapter III. The ECME algorithm is coded and run until the convergence maximum error 0.0000001 is satisfied or until the maximum number of iterations 3000 is reached. A common problem with this method is that the EM type algorithm may lead to a local maximum and one feasible solution to find the global maximum is to try many different initial values. Therefore, the procedure described by Basso et al. (2010) is adopted to ensure that initial values are not far from the real parameter values.

4.4.1 Determination of optimal model

To select the most appropriate model from normal, skew-normal and skew-t mixture models, the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Integrated Completed Likelihood Criterion (ICL) are computed for each mixture model. AIC and BIC have the same form $-2LL + \gamma c_n$, where LL is the log-likelihood value, γ is the number of free parameters to be estimated and c_n is the penalty term with a positive value.

The value of c_n is defined depending on the selected criterion. For AIC and BIC, c_n equals 2 and $\log(n)$ respectively, where n is the number of observations. The ICL criterion approximated from a BIC-like approximation is defined as $-2LL^* + \gamma \log(n)$, where LL^* is the integrated log-likelihood. It is known that BIC is more conservative than AIC. In the density estimation context, BIC is a reliable tool for comparing mixture models. When choosing the form of the model, using BIC as the criterion usually results in a good fit of data. If the finite mixture model is correctly specified, BIC is known to be consistent. On the other hand, if the concern of mixture modeling is cluster analysis, ICL criterion is preferred over BIC when selecting the optimal number of components g, because BIC may overestimate the number of components (Biernacki et al., 2000). In particular, BIC is likely to be imprecise in identifying the correct size of the clusters when component densities of mixture model are not specified correctly. The ICL criterion includes an additional entropy term which favors well-separated clusters (Biernacki et al., 2000).

Bold values in Table 4.1 report the smallest AIC, BIC among three mixture models. Smaller AIC and BIC values indicate a better overall fit. Based on the results, the skew-t mixture model is selected as the best one for g = 2, 3, 5, 6. For g = 4, the skew-normal mixture model is slightly better than the skew-t mixture model in terms of AIC and BIC values. Upon comparison of three mixture models, we find that the skew-normal and skew-t mixture models both show a much better fitting result than the normal mixture model; the skew-t mixture model has the smallest AIC and BIC values except when g equals 4. The computation times for each model are shown in Table 4.1. Compared with the normal mixture model, the skew-normal mixture model can significantly improve the goodness of fit of speed data while the increase in computational effort is not remarkable. Given this advantage, the skew-normal mixture model can be used as an alternative to the skew-t mixture model if the computation time is limited. And the skew-t mixture model can achieve the best fitting result at the cost of more computation time.

Another important criterion considered for model assessment is the Kolmogorov-Smirnov's (K-S) goodness of fit test (Lin et al., 2007). We performed K-S tests to validate the above three mixture models. The statistics D and p-value for K-S tests are summarized in Table 4.2. Note that in a K-S test, given a sufficiently large sample, a small and non-notable statistics D can be found to be statistically significant. For normal, skew-normal and skew-t mixture models, normal and skew-normal model with 2 components are rejected and none of skew-t mixture models is rejected when the significance level is 0.01. Thus, it also suggests that speed data can be better described by a mixture of skew-t distributions.

In summary, the skew-t mixture model outperforms the other two mixture models based on AIC, BIC and K-S test results. We select the skew-t mixture model as the best one and use it to determine the number of components. The parameter estimation results for the skew-t mixture distribution are provided in Table 4.3.

Table 4.1 Computed AIC, BIC and ICL values for three mixture models

g = 2	Normal	Skew-normal	Skew-t
AIC	232936.8	230936	**230223.5**
BIC	232978	230977.1	**230264.7**
ICL	234836.4	233345.5	**231732.3**
Time*	1 min	4 mins	45 mins
g = 3	Normal	Skew-normal	Skew-t
AIC	230254.7	229819.3	**229811.7**
BIC	230320.6	229885.2	**229877.6**
ICL	235846.5	242316.9	**235082.6**
Time*	1 min	6 mins	63 mins

Table 4.1 Continued

g = 4	Normal	Skew-normal	Skew-t
AIC	229921.4	**229801.9**	229802
BIC	230012	**229892.5**	229892.6
ICL	**239894**	256410.1	250663.8
Time*	4 mins	8 mins	363 mins
g = 5	Normal	Skew-normal	Skew-t
AIC	229836.3	229745	**229740.6**
BIC	229951.6	229860.4	**229855.9**
ICL	251178.7	**247112.5**	251844.7
Time*	8 mins	22 mins	438 mins
g = 6	Normal	Skew-normal	Skew-t
AIC	229809.1	229786.7	**229746**
BIC	229949.2	229926.7	**229886**
ICL	257663.9	**243317.1**	245020.3
Time*	18 mins	32 mins	518 mins

∗ These experiments were performed on a desktop with Core 2 Duo processor E8500 running at 3.16 GHz and 4 GB RAM.

Table 4.2 The K-S test results for three mixture models

No. of components	Normal		Skew-normal		Skew-t	
	D	p-value	D	p-value	D	p-value
g = 2	0.0275	0.0000	0.0220	0.0000	0.0146	0.0109
g = 3	0.0117	0.04242	0.0074	0.4796	0.0074	0.4825
g = 4	0.009	0.2055	0.0072	0.5016	0.0071	0.5141
g = 5	0.007	0.5038	0.0069	0.5444	0.0070	0.5256
g = 6	0.0067	0.5583	0.0073	0.4894	0.0067	0.5764

4.4.2 Selecting the number of components

It is quite a challenge to determine the optimal number of components in finite mixture models. Currently, available methods include reversible jump MCMC and model choice criteria. For skew-t mixture models, the implementation of reversible jump MCMC turns out to be very complicated and computation of marginal likelihoods remains an issue. Thus, we adopted the model choice criteria. As mentioned before, AIC tends to select too many components and BIC overrates the number of components if the component densities are misspecified. ICL criterion seems to provide a reliable estimate of g for real data (Biernacki et al., 2000). Thus, ICL values reported in Table 4.1 are used to determine the optimal number of components. Based on ICL criterion, g = 2 is chosen for the skew-t mixture model. Previously, Park et al. (2010) explored the data with a normal mixture model and selected the optimal number of components g = 4. To provide further insight into the pattern of mixture, we fit the speed distribution with a 2-component skew-t mixture model and a 4-component normal mixture model.

The mixture density as well as each component-wise density for the 2-component skew-t and 4-component normal mixture distributions are displayed in Figure 4.1 and Figure 4.2, respectively. Based on the graphical visualization, both 2-component skew-t and 4-component normal mixture models fit the 24-hour speed distribution very well. However, as shown in these figures, the bimodality of the speed distribution suggests the presence of 2 different speed groups. One skew-t distribution can adequately capture the skewness and kurtosis present in one cluster; by contrast, two normal mixtures are needed to accommodate the skewness and kurtosis of one speed group. It is observed in Figure 4.1 that cluster 1 is composed of speed data from group 1 and cluster 2 consists of speed data from group 2. Since group 1 and group 2 represent distinct traffic flow characteristics, this verifies that traffic flow condition is the main cause for heterogeneity in this 24-hour speed data. On the other hand, no clear interpretation can be made regarding different flow conditions if a 4-component normal mixture model is used.

To summarize, the skew-t mixture model classified vehicle speed into 2 clusters. Component 1 (high speed cluster) includes vehicles in uncongested traffic condition and a large portion of vehicles in transition flow condition. Component 2 (low speed cluster) has a large variance and represents vehicles in congested traffic condition and a small portion of vehicles in transition flow condition.

Table 4.3 Parameter estimation results for the Skew-t mixture distribution

Component	Parameters	1	2	3	4	5	6
g=2	ξ	101.71	6.96				
	σ^2	79.01	491.72				
	λ	-1.07	8.06				
	nu*	3.59	3.59				
	η	0.85	0.15				
g=3	ξ	88.21	93.92	7.27			
	σ^2	298.74	55.04	363.48			
	λ	-1.08	0.72	6.09			
	nu*	9.33	9.33	9.33			
	η	0.14	0.73	0.13			
g=4	ξ	78.36	93.66	7.23	99.78		
	σ^2	254.36	85.60	375.22	90.90		
	λ	-2.33	1.92	6.09	-1.52		
	nu*	15.05	15.05	15.05	15.05		
	η	0.07	0.39	0.13	0.41		

Table 4.3 Continued

g=5	ξ	40.41	7.67	93.05	71.68	100.58	
	σ^2	41.52	294.79	99.96	88.97	100.36	
	λ	2.98	4.97	2.49	-0.50	-1.59	
	nu*	20.43	20.43	20.43	20.43	20.43	
	η	0.01	0.12	0.35	0.06	0.45	
g=6	ξ	7.85	93.35	99.59	91.89	38.55	70.93
	σ^2	270.45	46.18	95.12	83.40	943.53	36.68
	λ	4.52	1.05	2.05	-0.90	9.99	-1.20
	nu*	100.00	100.00	100.00	100.00	100.00	100.00
	η	0.12	0.54	0.11	0.17	0.05	0.02

* Kurtosis parameter

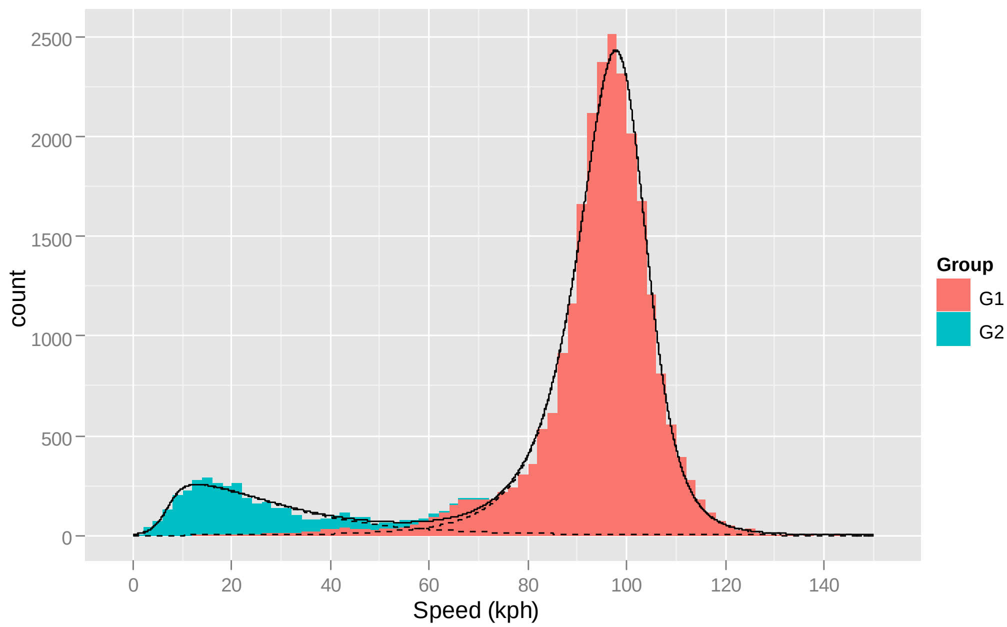


Figure 4.1 The fitted mixture model for 2-component skew-t distribution.

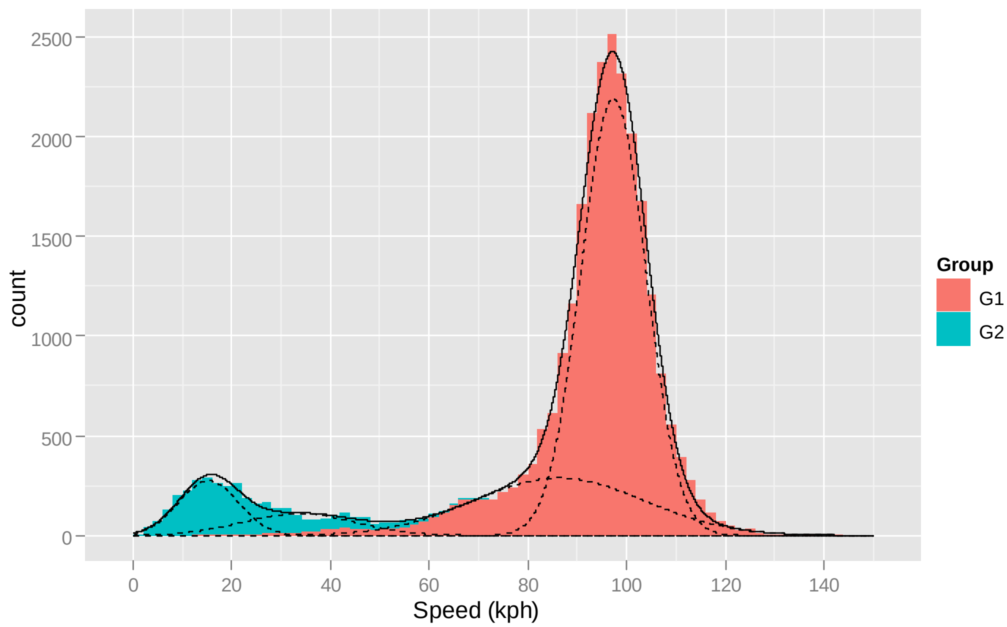


Figure 4.2 The mixture model for 4-component normal distribution.

4.5 Summary

This chapter has shown that skew-t distributions are useful for fitting the distribution of speed data. It is observed that for heterogeneous traffic flow condition, the flexibility of bimodal distribution causes problems when normal mixture models are used. The skew-t distributions are preferred component densities because they can capture skewness and excess kurtosis themselves. The finite mixture of skew-t distributions can significantly improve the goodness of fit of speed data.

CHAPTER V

METHODOLOGY II: MULTIVARIATE MIXTURE MODELING OF FREEWAY SPEED AND HEADWAY DATA

5.1 Introduction

The Farlie-Gumbel-Morgenstern (FGM) approach is applied to construct a bivariate distribution for describing the characteristics of speed and headway simultaneously. The FGM approach is classic and straightforward to use and its performance relies on the description accuracy of speed and headway distributions, called marginal distributions, as well as the correlation between speed and headway. In selecting the marginal distributions, we model the speed distribution using normal, skew-normal and skew-t mixture models and the headway distribution using gamma, lognormal and log-logistic models. This chapter shows that the constructed bivariate distribution can provide a satisfactory fit to the speed and headway distribution.

5.2 Methodology

5.2.1 Farlie-Gumbel-Morgenstern Approach

There are many methods of constructing bivariate distributions (see Bhat and Eluru's paper (2009) for an overview of the copula approaches). The FGM is an intuitive and natural way to construct the joint distribution function based on the marginal cumulative distribution functions (CDF) (Erdem and Shi, 2011). The joint CDF of a bivariate distribution constructed by the FGM approach can be described as follows:

$$H(x,y)=F(x)G(y)[1+a(1-F(x))(1-G(y))] \quad (5.1)$$

where H(x,y) represents the CDF of a bivariate distribution, F(x) is the marginal CDF of the first variable (i.e., the vehicle speed), G(y) denotes the marginal CDF of the second variable (i.e., the headway), and a is an association parameter. For absolutely continuous marginal distributions, we need $|a|\le 1$ (Schucany et al., 1978).

The joint probability density function (PDF) of a bivariate distribution can be obtained by a direct differentiation of H(x,y) and the joint density is:

$$h(x,y)=f(x)g(y)[1+a(2F(x)-1)(2G(y)-1)] \quad (5.2)$$

where h(x,y) represents the PDF of a bivariate distribution, f(x) is the marginal PDF of the first variable (i.e., the vehicle speed), and g(y) denotes the marginal PDF of the second variable (i.e., the headway).

The FGM was originally introduced by Morgenstern for Cauchy marginals and investigated by Gumbel for exponential marginals, and later generalized to arbitrary functions by Farlie. This approach has the limitation that only if the correlation of two variables is weak, the FGM can provide an effective way for constructing a bivariate distribution. The correlation structure of FGM bivariate distributions has been investigated for various continuous marginals such as uniform, normal, exponential, gamma and Laplace distributions. Schucany et al. (1978) showed that the correlation coefficient between X and Y can never exceed 1/3. Moreover, Schucany et al. (1978)

also indicated that regardless of the values of the parameters of the marginal distributions, the maximum correlation coefficient for identical normal marginals is $\frac{1}{\pi}$, $\frac{1}{4}$ for identical exponential marginals and 0.281 for the identical Laplace and gamma marginals (with shape parameter equal to 2). The weak dependence generated by the FGM family prompted some researchers to investigate the modifications of the FGM family. Kotz and Johnson (1977) and Huang and Kotz (1999) have proposed iterated FGM distributions to accommodate higher dependence. Although the iterated FGM distributions can allow a higher dependence, considering the weak dependence between speed and headway and the complexity of iterated FGM distributions, the FGM approach is used to generate bivariate distributions in this study.

To validate the applicability of the proposed approach, we examine the correlation structures of the bivariate FGM family with marginal distributions specified in the following section. The basic measure of dependence between two variables is the covariance. From equation (5.2), the covariance function $Cov(x,y)$ can be obtained as:

$$Cov(x,y) = a\int x[2F(x)-1]f(x)dx\int y[2G(y)-1]g(y)dy \tag{5.3}$$

The Pearson's product-moment correlation coefficient ρ is defined as:

$$\rho = \frac{Cov(x,y)}{\sigma_x \sigma_y} \tag{5.4}$$

where σ_x and σ_y are the standard deviations of variables x and y.

The correlation coefficient ρ can then be derived as follows (for a proof, see Takeuchi, 2010):

$$\rho = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{a}{\sigma_x \sigma_y} \int x[2F(x)-1] f(x)dx \int y[2G(y)-1] g(y)dy \tag{5.5}$$

From equation (5.5) and the fact that $|a| \leq 1$, the correlation coefficient ρ of a bivariate FGM distribution is hence limited to the following range

$$|\rho| \leq \frac{1}{\sigma_x \sigma_y} \int x[2F(x)-1] f(x)dx \int y[2G(y)-1] g(y)dy \tag{5.6}$$

Equation (5.6) can be used to determine the maximum correlation coefficient ρ for the FGM family with prescribed marginal distributions. Although the PDFs of marginal distributions are given in the following section, it is very difficult to integrate equation (5.6) for some distributions (i.e., skew-normal and skew-t mixture models). Thus, to overcome the difficulties of the integration, we first determine the values of parameters of the marginal distributions by fitting the distributions to speed and headway data, respectively. Then, the numerical integration method is applied to equation (5.6) and the range of correlation coefficient ρ can then be estimated.

5.2.2 Marginal Distribution

In this study, if the characteristics of speed data are heterogeneous (or bimodal distribution), the mixture models of normal, skew-normal and skew-t are used to capture excess skewness, kurtosis and bimodality present in speed distribution. The mixture model is widely used in modeling bimodal speed distribution to account for the heterogeneity. To identify the values of parameters in the selected distributions, the Expectation/Conditional Maximization Either (ECME) algorithm is chosen to estimate the parameters of normal, skew-normal and skew-t mixture models in this study. For more details about the parameter estimation, interested readers could consult Zou and Zhang (2011). All statistical analyses were carried out in Software R (2006).

For the headway model, three commonly used single distribution models are investigated: gamma, lognormal and log-logistic distributions. After determining the statistical distribution governing headway, the next step is to estimate the parameters of three proposed models. We also used the maximum likelihood estimation approach to estimate the parameters. The three headway models are briefly introduced as follows.

The density function of a gamma distribution for headway can be written as:

$$f(x \mid \alpha, \beta) = \frac{x^{\alpha-1} e^{(-x/\beta)}}{\Gamma(\alpha)\beta^{\alpha}} \tag{5.7}$$

$$E(x) = \alpha\beta \tag{5.8}$$

$$Var(x) = \alpha\beta^2 \tag{5.9}$$

where α is the shape parameter, β is the scale parameter, and $\Gamma(x)$ is the gamma function.

The lognormal distribution has the following probability density function:

$$f(x \mid \mu, \sigma^2) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \tag{5.10}$$

$$E(x) = \exp(\mu + \frac{\sigma^2}{2}) \tag{5.11}$$

$$Var(x) = (\exp(\sigma^2) - 1)\exp(2\mu + \sigma^2) \tag{5.12}$$

where μ and σ are the mean and standard deviation, respectively, of the variable's natural logarithm.

The density function of a log-logistic distribution can be formulated as follows:

$$f(x \mid \alpha, \beta) = \frac{(\alpha / \beta)(x / \beta)^{\alpha - 1}}{[1 + (x / \beta)^{\alpha}]^2} \tag{5.13}$$

$$E(x) = \frac{\beta\pi / \alpha}{\sin(\pi / \alpha)} \quad \text{for} \quad \alpha > 1 \tag{5.14}$$

$$Var(x) = \beta^2 (2(\pi / \alpha) / \sin(2\pi / \alpha) - (\pi / \alpha)^2 / \sin^2(\pi / \alpha)) \quad \text{for} \quad \alpha > 2 \tag{5.15}$$

where α is the shape parameter, and β is the scale parameter and also the median of the distribution.

5.2.3 Goodness of Fit Statistics

To evaluate the goodness of fit of the selected distributions for speed and headway data as well as the bivariate distribution, the R^2 and RMSE statistics are used in this study. R^2 statistic is a bin-specific test and measures the strength of linear relationship between the expected and observed frequencies of the bins. The common definition of the R^2 is

$$R^2 = 1 - \frac{SS_{err}}{SS_{tot}} \tag{5.16}$$

where SS_{err} represents the sum of squares of the residuals and measures the total difference between the observed and expected frequency for all of the bins, and SS_{tot} denotes the total sum of squares and assesses the total difference between the observed and average frequencies for all bins. R^2 statistic ranges from 0 to 1 and higher R^2 values indicate a better fit.

The RMSE statistic is also bin-specific and has the following form:

$$RMSE = \sqrt{\frac{SS_{err}}{N_T}} \tag{5.17}$$

where SS_{err} represents the sum of squares of residuals, and N_T is the total number of bins. Unlike the R^2 statistic, higher RMSE values indicate a poorer fit. R^2 can be considered to be a more precise goodness of fit metric, because it uses the number of non-empty bins in the equation, where the RMSE value makes use of the total number of bins as a parameter in the equation. Note that when calculating the R^2 and RMSE statistics for the bivariate distribution, SS_{err} reflects the total difference between the observed and expected frequency for all of the two-dimensional bins, and N_T is the total number of two-dimensional bins. For speed, the bin size of R^2 metric is fixed at 2 kph, whereas for headway, the bin size is specified as 1 second. The RMSE metric uses the same bin size.

5.3 Results and discussions

5.3.1 Marginal Models for Speed and Headway

Table 5.1 reports the estimated parameters, R^2 and RMSE values of different speed models. Higher R^2 and lower RMSE values indicate a better overall fit. It can be seen that all speed models have high R^2 values. For the 24 hour speed data, the 4-component normal mixture model is slightly better than the 2-component skew-t mixture model in term of goodness of fit index. However, previous studies (Zou and Zhang, 2011) showed that the 2-component skew-t mixture model can better account for heterogeneity in the speed data. The histogram of speed data is given in Figure 2 (a). Based on the graphical visualization, all speed models fit the 24-hour speed data well. In the meantime, the headway data were examined using gamma, lognormal and log-logistic models. The performance of headway models is not consistent. The estimated parameters, R^2 and RMSE values are shown in Table 5.2. Based on the results, the log-logistic model has the highest R^2 and lowest RMSE values and the gamma model provides the least satisfactory fitting performance. It is speculated that the gamma model may not be able to capture the sharp peak present in the headway histogram. The histogram of headway data is shown in Figure 2 (b). Note that speed and headway histograms have different total number of bins (N_T), respectively. And this explains that although the 4-component normal mixture model for speed and the lognormal model for headway have almost the same R^2 values, the corresponding RMSE values differ significantly.

Table 5.1 Parameter Estimation Results, R2 and RMSE Values for Speed Models

Speed model	Parameter	group1	group2	group3	group4	R^2	RMSE
4-component normal	mu	15.37	33.60	86.39	97.36	0.982	81.90
	sigma	6.00	11.39	17.28	6.59		
	w	0.07	0.06	0.22	0.65		
2-component skew-normal	mu	5.46	102.92			0.969	106.90
	sigma	48.50	10.39				
	shape	20.15	-1.26				
	w	0.21	0.79				
2-component skew-t	mu	6.95	101.71			0.979	87.65
	sigma	22.22	8.89				
	shape	8.09	-1.07				
	nu	3.59	3.59				
	w	0.15	0.85				

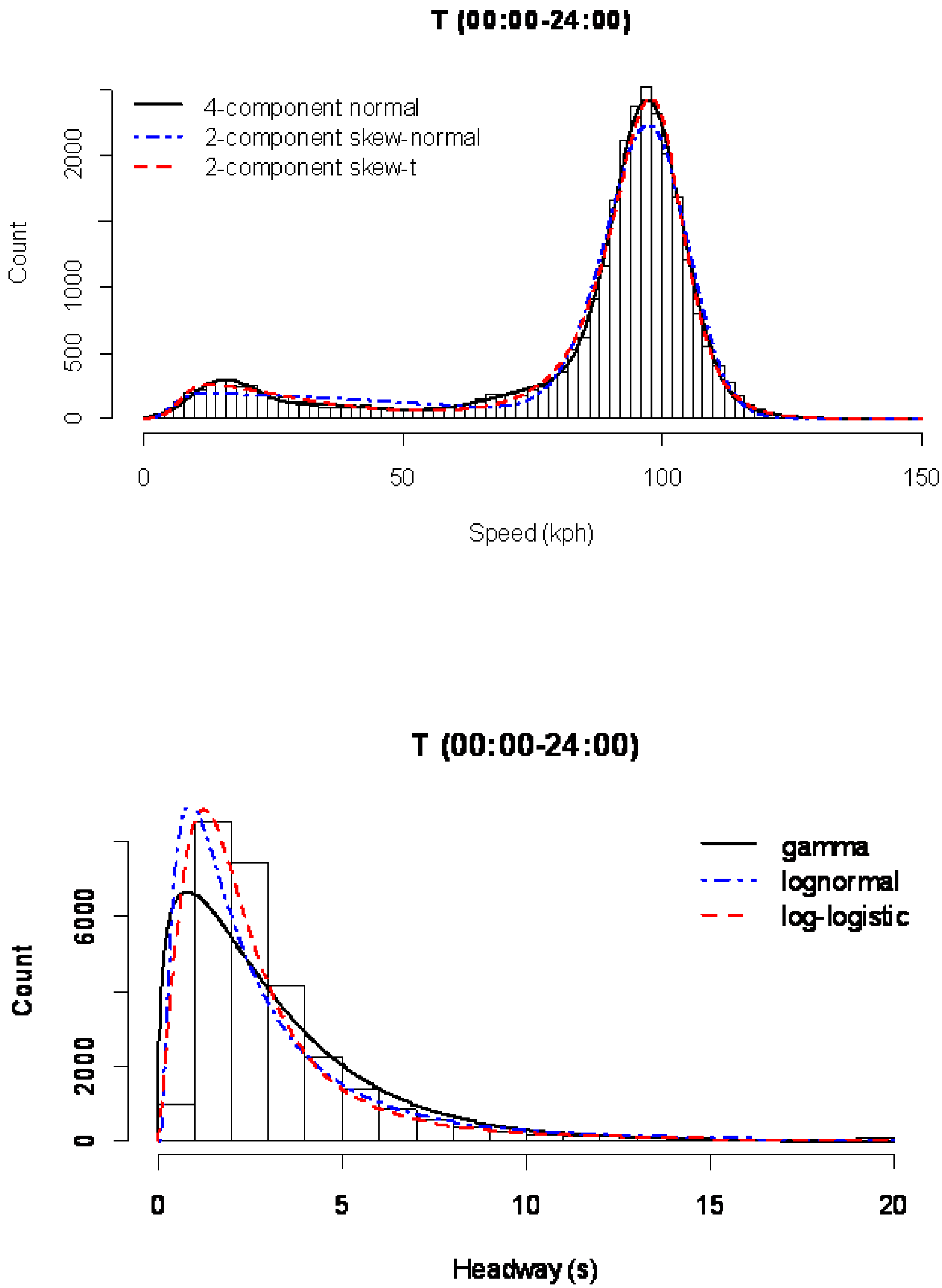


Figure 5.1 24-hour speed and headway histograms (a) speed (b) headway.

Table 5.2 Parameter Estimation Results, R^2 and RMSE Values for Headway Models

Headway model	Parameter		R^2	RMSE
gamma	shape	1.34	0.853	528.53
	scale	2.31		
lognormal	mu	0.71	0.981	191.71
	sigma	0.93		
log-logistic	shape	2.05	0.999	40.47
	scale	2.07		

5.3.2 Correlation between Speed and Headway

As mentioned in the previous section, the FGM approach has the limitation that the correlation coefficients between the two variables should not exceed 1/3. For each combination of marginal distributions of speed and headway, the maximum correlation coefficient is determined by numerically integrating equation (5.6) and the results are provided in Table 5.3. The Pearson correlation coefficient is calculated in this study to verify the condition that it is smaller than the maximum correlation coefficients. The Pearson correlation coefficient is -0.0541. Since the Pearson correlation coefficient is well below the maximum correlation coefficients provided in Table 5.3, the results show that the weak correlation coefficient between speed and headway renders the FGM models suitable for our research on bivariate distributions.

Table 5.3 Maximum Correlation Coefficients for Each Marginal Distribution

	gamma	lognormal	log-logistic
4-component normal	0.246	0.197	0.105
2-component skew-normal	0.248	0.199	0.106
2-component skew-t	0.240	0.193	0.103

One perspective to assess the correlation between speed and headway is to study the headway variation at different speed levels. In the study of Brackstone et al. (2009), it is revealed that there is a limited dependence of following headway on speed; the most successful relationship fit of headway and speed is inverse relationship. They also suggested that driver response can be split into two regimes. The following time headway is approximately constant when speed is above 15 m/s (54 kph) in the first regime. In the second regime when speed is below 15 m/s (54 kph), the following time headway increases as the speed decreases. Following the same procedure used in Brackstone et al.'s study (2009), we divide the speed data (kph) into seven groups, and the average headway for each speed group is given in Table 5.4. From Figure 1 (a), since speed values in speed groups 1 through 4 are under 72 kph, it can be seen that speed groups 1 through 4 consist of 4944 vehicles with recorded speed values observed under congested traffic conditions. Thus, it is reasonable to consider headway values corresponding to those speed data in speed groups 1 through 4 as following headways. It is observed that within these four speed groups, the mean of following headway increases as the speed decreases, which is consistent with the findings in the study by Brackstone et al. (2009). On the other hand, for speed groups 5 through 7, 22975 vehicles are observed in off peak periods (uncongested traffic condition). Considering few vehicles on the road, vehicles can easily reach the free flow speed. As the speed grows higher, fewer vehicles are on the road and it is expected to see that the headway increases accordingly.

Table 5.4 Summary Statistics of Headway for Each Speed Group

Speed group (kph)	Group 1 0-18	Group 2 18-36	Group 3 36-54	Group 4 54-72	Group 5 72-90	Group 6 90-108	Group 7 108+
Mean (s)	5.67	3.17	2.27	1.9	2.19	3.13	4.05
Min. (s)	1	0	0	0	0	0	0
Max. (s)	48	17	12	11	42	68	76
No. of vehicles	1501	1484	757	1202	4548	16698	1729

In order to further examine the correlation between speed and headway on IH-35, we study the determinants of the individual vehicle speed and the individual vehicle headway over a 24-hour period. It is important for us to note that the vehicle speed and the vehicle headway will affect each other as well. Because of this interrelationship, formulating separate ordinary least squares regression models of speed and headway would result in inefficient parameter estimates. To address this issue, Seemingly Unrelated Regressions (SUR) estimation is used to account for the direct correlation between speed and headway. Adopting a similar equation system used by Martchouk et al. (2011), the speed and headway are determined in a simultaneous equation system formalized as follows:

$$Speed = C_S + \beta_1 \times Headway + \sum_{j=2}^{n} \beta_j X_j + \varepsilon_S \quad (5.18)$$

$$Headway = C_H + \alpha_1 \times Speed + \sum_{i=2}^{m} \alpha_i Y_i + \varepsilon_H \quad (5.19)$$

where Speed is the individual vehicle speed collected over a 24-hour period, Headway represents the corresponding individual vehicle headway, β_j, for j=1,…,n, are estimated parameters for speed equation, α_i, for i=1,…,m, are estimated parameters for headway equation, X_j and Y_i are independent variables, C_S and C_H are constants, and ε_S and ε_H are error terms. For the parameter estimation, the SUR model is estimated using systemfit package in the software R.

The SUR estimation results for speed and headway are provided in Tables 5.5 and 5.6, respectively. In general, the speed and headway models fit the data well and the coefficient estimates have plausible sign and are all statistical significant at the 95% level. The adjusted R^2 for the vehicle speed model is 0.8387 and 0.2638 for the model of vehicle headway.

In the model of individual vehicle speed (Table 5.5), the coefficient of variable headway is -0.0727 which indicates that higher vehicle headway slightly decreases vehicle speed.

This finding is consistent with the calculated Pearson correlation coefficient which is -0.0541. For other variables, the increase of variable vehicle length also results in slower vehicle operating speed. The AM and PM peak-hour variables can reduce the vehicle speed. Since IH-35 experiences worst traffic congestion condition during afternoon peak period, the vehicle speed drops significantly in the PM peak-hour. At night, drivers tend to drive faster because fewer vehicles are on the road. In the model of individual vehicle headway (Table 5.6), the coefficient of variable speed is -0.0058, which is also consistent with the Pearson correlation coefficient and shows that higher vehicle speed slightly decreases vehicle headway. As expected, longer vehicle length and preceding vehicle length result in larger vehicle headway, which is consistent with the finding by Ye and Zhang (2009). The reason is that trucks and buses generally accelerate and decelerate more slowly than passenger cars. During the PM peak hours, vehicle headway increases because most vehicles drove at 10-30 kph. Meanwhile, during the AM peak hours, vehicle headway decreases because the flow almost reaches the capacity of the road and vehicles keep a minimum following headway. For the variable night time, it leads to higher vehicle headway. Note that based on the provided t-statistic values, the effects of variables headway (in Table 5.5) and speed (in Table 5.6) are found to be significant at the 95% level. The modeling results from the SUR model suggest that there is an inverse relationship between speed and headway. Overall, based on the Pearson correlation coefficient, it can be seen that there is a weak dependence between speed and headway, and this weak dependence renders the FGM models suitable for constructing the joint distribution.

Table 5.5 Seemingly Unrelated Regression Model of Individual Vehicle Speed (in kph)

Variable	Coefficient Estimate	Standard Error	t-statistic
Constant	97.9186	0.1207	811.0385
Headway (second)	-0.0727	0.0190	-3.8383
Vehicle length (ft)	-0.1827	0.0139	-13.1477
AM peak-hour indicator (1 if time between 07:00-08:30, 0 otherwise)	-22.5995	0.2103	-107.4438
PM peak-hour indicator (1 if time between 15:30-19:30, 0 otherwise)	-73.0294	0.1992	-366.6260
Night-time indicator (1 if time between 00:00-05:00, 0 otherwise)*	4.2229	0.3088	13.6765
Number of Observations	27919		
Adjusted R-Squared	0.8387		

* Due to the consistently heavy traffic on IH-35, the night time interval was selected from 00:00 to 05:00.

Table 5.6 Seemingly Unrelated Regression Model of Individual Vehicle Headway (in seconds)

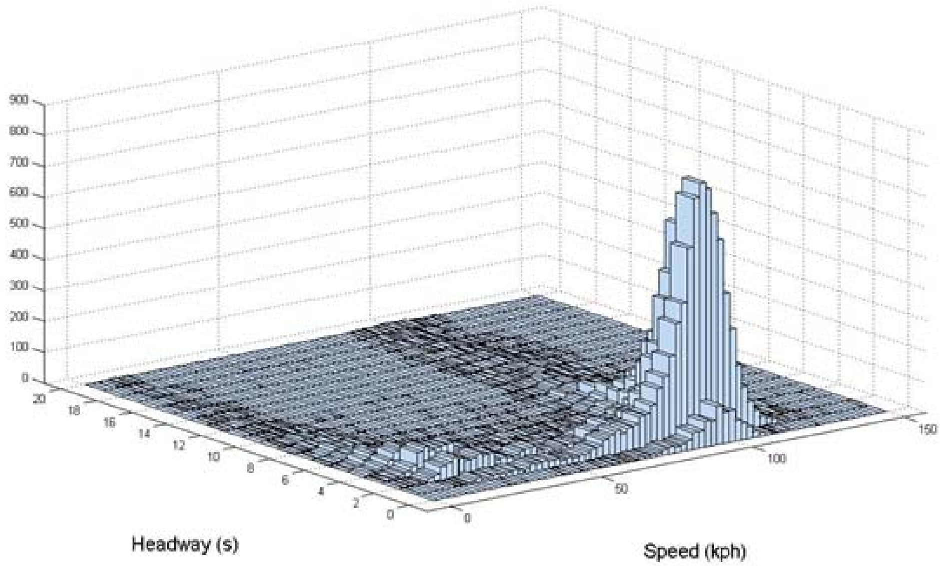
Variable	Coefficient Estimate	Standard Error	t-statistic
Constant	2.3495	0.1899	12.3696
Speed (kph)	-0.0058	0.0019	-3.0708
Vehicle length (ft)	0.0353	0.0044	8.0507
AM peak-hour indicator (1 if time between 07:00-08:30, 0 otherwise)	-0.7403	0.0785	-9.4338
PM peak-hour indicator (1 if time between 15:30-19:30, 0 otherwise)	1.2166	0.1508	8.0675
Night-time indicator (1 if time between 00:00-05:00, 0 otherwise)	7.5640	0.0865	87.4167
Preceding vehicle length (ft)	0.0760	0.0044	17.3599
Number of Observations	27919		
Adjusted R-Squared	0.2638		

5.3.3 Bivariate Distributions

Bivariate histograms based on the speed and headway distributions need to be created for the computation of the R^2 and RMSE values. After creating a two-dimensional binning structure, the bivariate probability density function needs to be integrated to obtain the expected frequencies for each bin. Table 5.7 provides the R^2 and RMSE values based on the bivariate distributions. In Table 5.7, nine R^2 and RMSE values are given for nine combinations of marginal distributions between speed and headway. The combination of the 4-component normal mixture distribution for speed and log-logistic distribution for headway yields the highest R^2 and lowest RMSE values. The combination of 2-component skew-normal mixture distribution for speed and gamma distribution for headway gives the smallest R^2 and largest RMSE values. For the purpose of brevity, we only give the bivariate histogram and the best fitted joint distribution in Figure 3 (a) and (b). It can be observed that the trend of the bivariate histogram profile is well captured by the FGM bivariate distribution.

Table 5.7 R^2 and RMSE Values for Bivariate Distributions

R^2	gamma	lognormal	log-logistic
4-component normal	0.798	0.925	0.949
2-component skew-normal	0.788	0.912	0.938
2-component skew-t	0.796	0.922	0.947
RMSE	gamma	lognormal	log-logistic
4-component normal	16.826	10.283	8.418
2-component skew-normal	17.25	11.074	9.331
2-component skew-t	16.901	10.425	8.59



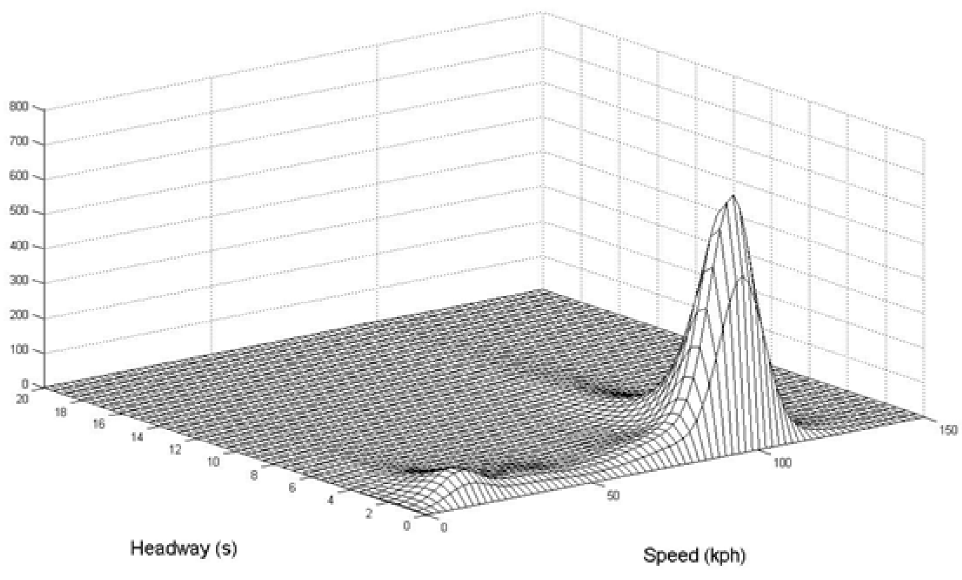


Figure 5.2 Bivariate speed and headway histogram and fitted distribution: (a) bivariate speed and headway histogram; (b) fitted bivariate distribution.

5.3.4 Discussion

The modeling results are very interesting and deserve further discussion. Although the bivariate distribution has been successfully constructed in this study, if we compare the R^2 values in Tables 5.1 and 5.2 with those in Table 5.7, it can be found that the goodness of fit index deteriorates slightly for bivariate distributions as compared with the one-dimensional counterpart. The accuracy of marginal distributions is critical when using the FGM approach to construct bivariate distributions, and satisfactory fitting performance of bivariate distribution can be only achieved when appropriate marginal distributions are selected. Note that the R^2 values for marginal distributions of speed and headway are always higher than the R^2 values for corresponding bivariate distributions. For example, the R^2 value of gamma distribution for headway is 0.853, and any bivariate distribution that includes the gamma distribution for headway has a R^2 value less than 0.8. Gamma distribution is the least performing marginal distribution for headway, and having gamma distribution as the marginal distribution for headway lead to the least performing bivariate distribution model. Therefore, it is necessary to provide the most appropriate marginal distributions for speed and headway in order to construct a satisfactory bivariate distribution using the FGM approach.

Traditionally, speed and headway are often not studied jointly in microscopic simulation models. As a result, the same headway distribution may be assumed for different speed levels and this assumption neglects the possible variability of headway distribution across speed values. To overcome this potential problem associated with the traditional approach, the constructed bivariate distribution in this study can be used to determine the headway distribution for given speed values, and vice versa. For example, given a specific value of speed, the headway distribution corresponding to that speed value can be derived based on equation (5.2). Thus, the proposed method can be used to generate vehicle speeds and vehicle arrival times simultaneously by considering the dependence between speed and headway.

There are several avenues for further work. First, since the applicability of the FGM approach is affected by the correlation of the two variables, some other existing methods to construct joint distributions such as marginal transformation method, the mixing and compounding methods and conditionally specified distributions (Balakrishnan and Lai, 2009) can be investigated. It would also be interesting to see the comparison of modeling results by using different bivariate distribution construction approaches. Second, realizing the effect of vehicle type on the dependence between speed and headway, it is useful to classify the traffic data based on vehicle types and construct bivariate distributions of speed and headway for each vehicle type separately (i.e., trucks and passenger cars). Third, since the speed and headway data are site dependent and different sites may have distinct traffic characteristics, more sites should be investigated to fully explore the relationship between speed and headway.

5.4 Conclusions

In this study, the FGM approach is successfully applied to construct the bivariate distribution for speed and headway. The important findings and recommendations can be summarized as follows. First, the modeling results based on empirical data indicate that there is a weak inverse relationship between the 24-hour speed and headway data, and the weak dependence renders the FGM approach suitable for constructing bivariate distributions. Second, the FGM is an effective approach to construct bivariate distributions and when using the FGM, it is necessary to provide the most appropriate marginal distributions for speed and headway in order to construct a satisfactory bivariate distribution. Overall, as mentioned in the discussion section, the proposed FGM approach can consider the dependence between speed and headway which is often neglected in traditional microscopic simulation models. Thus, this study provides a framework for developing microscopic simulation models to generate vehicle speeds and vehicle arrival times simultaneously. In addition, the statistical methods used to establish the relationship between speed and headway can be further applied to speed and headway data from multiple areas and locations.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

Traditionally, traffic variables (speed and headway) are often not studied jointly in microscopic simulation models. One important flaw associated with the traditional approach is that the simulated samples based on the independence assumption usually fail to consider the empirical dependence between traffic variables. To overcome this potential problem associated with the traditional approach, it is necessary to construct bivariate distributions to model vehicle speed and headway simultaneously.

The research first examined the dependence structure between speed and headway using three measures of dependence (i.e., Pearson correlation coefficient, Spearman's rho and Kendall's tau). The research proposed the skew-t mixture models to capture heterogeneity present in speed distribution. To develop a bivariate distribution for capturing the dependence, the FGM method was applied to the 24-hour speed and headway data.

6.2 Conclusions

Based on the modeling results from this research, we drew some important conclusions, which are listed as follows:

1. The proposed skew-t mixture models can reasonable account for heterogeneity problem in freeway vehicle speed data. Finite mixture of skew-t distributions can significantly improve the goodness of fit of speed data. The methodology developed in this research can be used in analyzing the characteristics of freeway speed data. Considering that many traffic analytical and simulation models use speed as an input for travel time and level of service determination, the developed models can generate more accurate speed value as the input and help improving the reliability of the analysis output.

2. There exists weak dependence between speed and headway. The dependence between speed and headway is strongest under the most congested traffic condition. The bivariate model can provide a satisfactory fit to the multimodal speed and headway distribution. The proposed methodology can overcome the correlation problem associated with the traditional approach.

6.3 Future Research

This research proposes the FGM approach to construct bivariate distributions to describe the characteristics of speed and headway, and there are some avenues for future work.

1. A better understanding of speed and headway distributions and its dependence structure can help operational analysis of a freeway facility. In future, since the speed and headway data are site dependent and different sites may have distinct traffic characteristics, multiple locations should be investigated to fully explore the relationship between speed and headway.

2. Traffic headway includes time headway and distance headway, which are closely related to each other and both vary depending on speed and traffic condition. Distance headway is also an important microscopic traffic variable and one influential factor in the car following model. Some studies have shown that there exists positive dependence between distance headway and speed. Thus, if the distance headway data is available in this study, we can further investigate the dependence structure between distance headway and speed. The findings from further analysis may contribute to the existing car following theory.

3. In some popular traffic simulation models (i.e., CORSIM, Sim Traffic and VISSIM), vehicles are usually generated on the basis of a certain headway distribution. CORSIM considers three types of vehicle entry headway generation distributions: uniform, normal and Erlang distributions. The negative exponential distribution is used in VISSIM and SimTraffic. The current simulation protocols in these microscopic traffic simulation models fail to consider the dependence between speed and headway. Thus, in the future, the bivariate distributions can be used in these traffic simulation models to generate more accurate speed and headway of entry vehicle.

REFERENCES

AZZALINI, A. 1985. A Class of Distributions Which Includes the Normal Ones. *Scandinavian Journal of Statistics,* 12**,** 171-178.

AZZALINI, A. & CAPITANIO, A. 2003. Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution. *Journal of the Royal Statistical Society Series B-Statistical Methodology,* 65**,** 367-389.

BASSO, R. M., LACHOS, V. H., CABRAL, C. R. B. & GHOSH, P. 2010. Robust mixture modeling based on scale mixtures of skew-normal distributions. *Computational Statistics & Data Analysis,* 54**,** 2926-2941.

BHAT, C. R. & ELURU, N. 2009. A copula-based approach to accommodate residential self-selection effects in travel behavior modeling. *Transportation Research Part B-Methodological,* 43**,** 749-765.

BIERNACKI, C., CELEUX, G. & GOVAERT, G. 2000. Assessing a mixture model for clustering with the integrated completed likelihood. *Pattern Analysis and Machine Intelligence, IEEE Transactions on,* 22**,** 719-725.

BRACKSTONE, M., WATERSON, B. & MCDONALD, M. 2009. Determinants of following headway in congested traffic. *Transportation Research Part F-Traffic Psychology and Behaviour,* 12**,** 131-142.

COWAN, R. J. 1975. Useful Headway Models. *Transportation Research,* 9**,** 371-375.

DEMPSTER, A. P., LAIRD, N. M. & RUBIN, D. B. 1977. Maximum Likelihood from Incomplete Data Via Em Algorithm. *Journal of the Royal Statistical Society Series B-Methodological,* 39**,** 1-38.

DEY, P. P. & CHANDRA, S. 2009. Desired Time Gap and Time Headway in Steady-State Car-Following on Two-Lane Roads. *Journal of Transportation Engineering-Asce,* 135**,** 687-693.

DEY, P. P., CHANDRA, S. & GANGOPADHAYA, S. 2006. Speed distribution curves under mixed traffic conditions. *Journal of Transportation Engineering-Asce,* 132**,** 475-481.

ERDEM, E. & SHI, J. 2011. Comparison of bivariate distribution construction approaches for analysing wind speed and direction data. *Wind Energy,* 14**,** 27-41.

FR HWIRTH-SCHNATTER, S. 2006. *Finite mixture and Markov switching models*.

GERLOUGH, D. L. & HUBER, M. J. 1976. Traffic flow theory.

HAIGHT, F. A. 1965. *Mathematical theories of traffic flow*.

HAIGHT, F. A. & MOSHER, W. W. 1962. A practical method for improving the accuracy of vehicular speed distribution measurements. *Highway Research Board Bulletin*.

HOOGENDOORN, S. P. & BOVY, P. H. 1998. New estimation technique for vehicle-type-specific headway distributions. *Transportation Research Record: Journal of the Transportation Research Board,* 1646**,** 18-28.

HUANG, J. & KOTZ, S. 1999. Modifications of the Farlie-Gumbel-Morgenstern distributions. A tough hill to climb. *Metrika,* 49**,** 135-145.

JOHNSON, N. L. & KOTZ, S. 1977. On some generalized farlie-gumbel-morgenstern distributions-II regression, correlation and further generalizations. *Communications in Statistics-Theory and Methods,* 6**,** 485-496.
JUN, J. 2010. Understanding the variability of speed distributions under mixed traffic conditions caused by holiday traffic. *Transportation Research Part C-Emerging Technologies,* 18**,** 599-610.
KO, J. & GUENSLER, R. L. Characterization of congestion based on speed distribution: a statistical approach using Gaussian mixture model. Transportation Research Board Annual Meeting, 2005. Citeseer.
LEONG, H. The distribution and trend of free speeds on two lane two way rural highways in New South Wales. Australian Road Research Board (ARRB) Conference, 4th, 1968, Melbourne, 1968.
LIN, T. I., LEE, J. C. & HSIEH, W. J. 2007. Robust mixture modeling using the skew t distribution. *Statistics and Computing,* 17**,** 81-92.
LUTTINEN, R. 1992. Statistical properties of vehicle time headways. *Transportation research record*.
LUTTINEN, R. T. 1999. Properties of Cowan's M3 headway distribution. *Transportation Research Record: Journal of the Transportation Research Board,* 1678**,** 189-196.
MARTCHOUK, M., MANNERING, F. & BULLOCK, D. 2010. Analysis of freeway travel time variability using Bluetooth detection. *Journal of Transportation Engineering,* 137**,** 697-704.
MCLEAN, J. Observed speed distributions and rural road traffic operations. Australian Road Research Board Conference Proc, 1979.
PARK, B. J., ZHANG, Y. L. & LORD, D. 2010. Bayesian mixture modeling approach to account for heterogeneity in speed data. *Transportation Research Part B-Methodological,* 44**,** 662-673.
SCHUCANY, W. R., PARR, W. C. & BOYER, J. E. 1978. Correlation Structure in Farlie-Gumbel-Morgenstern Distributions. *Biometrika,* 65**,** 650-653.
TAIEB-MAIMON, M. & SHINAR, D. 2001. Minimum and comfortable driving headways: Reality versus perception. *Human Factors: The Journal of the Human Factors and Ergonomics Society,* 43**,** 159-172.
TAKEUCHI, T. T. 2010. Constructing a bivariate distribution function with given marginals and correlation: application to the galaxy luminosity function. *Monthly Notices of the Royal Astronomical Society,* 406**,** 1830-1840.
WASIELEWSKI, P. 1979. Car-following headways on freeways interpreted by the semi-Poisson headway distribution model. *Transportation Science,* 13**,** 36-55.
WINSUM, W. V. & HEINO, A. 1996. Choice of time-headway in car-following and the role of time-to-collision information in braking. *Ergonomics,* 39**,** 579-592.
YE, F. & ZHANG, Y. 2009. Vehicle type-specific headway analysis using freeway traffic data. *Transportation Research Record: Journal of the Transportation Research Board,* 2124**,** 222-230.

YE, Z., ZHANG, Y. & MIDDLETON, D. R. 2006. Unscented Kalman filter method for speed estimation using single loop detector data. *Transportation Research Record: Journal of the Transportation Research Board,* 1968**,** 117-125.

YIN, S. C., LI, Z. H., ZHANG, Y., YAO, D., SU, Y. L. & LI, L. 2009. Headway Distribution Modeling with Regard to Traffic Status. *2009 Ieee Intelligent Vehicles Symposium, Vols 1 and 2***,** 1057-1062.

ZHANG, G., WANG, Y., WEI, H. & CHEN, Y. 2007. Examining headway distribution models with urban freeway loop event data. *Transportation Research Record: Journal of the Transportation Research Board,* 1999**,** 141-149.

ZHANG, Y., XIE, Y. & YE, Z. Estimation of large truck volume using single loop detector data. Transportation Research Board 87th Annual Meeting, 2008.

ZOU, Y. & ZHANG, Y. 2011. Use of skew-normal and skew-t distributions for mixture modeling of freeway speed data. *Transportation Research Record: Journal of the Transportation Research Board,* 2260**,** 67-75.